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Stochastic Power Control for Time-Varying Fading Wireless Communication Networks

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Abstract—**The performance of stochastic optimal power control for time-varying fading channels, in which the evolution of the dynamical channel is described by stochastic differential equations, is determined. Both long-term and short-term time-varying models for fading wireless channels are considered. These models essentially capture the spatio-temporal variations of wireless channels. The solution of the stochastic optimal control is obtained through path-wise optimization, which is solved by linear programming using a predictable power control strategy. The algorithm can be implemented using an iterative numerical scheme. The performance of the algorithm is interference or outage probability. The algorithm can be used as long as the time duration for successive adjustments of transmitter powers is less than the coherence time of the channel.**

I. Introduction

POWER control is important to improve performance of wireless communication systems. Most of the research that has been done in this field deals mainly with time invariant wireless channel models. But in reality, wireless channels are time varying due to the relative motion between transmitters and receivers and temporal variations of the propagation environment [1].

The power allocation problem has been studied extensively as an eigenvalue problem for non-negative matrices in [2] and [3], resulting in iterative power control algorithms that converge each user's power to the minimum power [4], [5], and in optimization based approaches [6]. Much of this previous work deals with random but time invariant channel models. In a stochastic framework, attempts at recognizing the time correlated nature of signals are made in [7], where blocking is defined via the sojourn time of global interference above a given level which, if sufficiently long, induces blocking. Downlink power control for fading channels is studied in [8] by a heavy traffic limit where averaging methods are used. Stochastic control approach for uplink lognormal fading channels is studied in [9], in which a bounded rate power adjustment model is proposed.

In contrast with these papers, the modeling and analysis of power control strategies investigated here employ wireless models which are time-varying and subject to fading. The random variables characterizing the instantaneous power in static channel models are generalized to dynamical models including random processes with time-varying statistics. The dynamics of the channel is captured by stochastic differential equations (SDE's). A stochastic power control algorithm (PCA) is applied to determine the optimal transmitted powers. The proposed PCA is based on predictable power control strategy (PPCS) that was first introduced in [10]. The PPCS algorithm is proven to be effectively applicable to such dynamical models for an optimal power control. The outage probability is used as a performance measure for the proposed algorithm. Also the iterative power control algorithm presented in [4] and [5] can be used to determine the optimal powers iteratively. This helps in allowing autonomous execution at the node or link level, requiring minimal usage of network communication resources for control signaling. Simulation results are provided comparing the performance of the proposed method, PPCS, with the performance of constant power transmits for both time-varying long-term fading (LTF) and short-term fading (STF) channels.

The paper is organized as follows. In Section II, time-varying LTF and STF channel models in which the evolution of the channel is described by SDE's are introduced. In Section III, the stochastic optimal power control model is proposed and the solution of the proposed algorithm is obtained through linear programming using PPCS. In Section IV, simulation results are presented for both LTF and STF. Finally Section V gives the conclusion of the work developed in this paper.

II. TIME VARYING FADING CHANNEL MODELS

A. Long Term Fading (LTF) Channel Model

The power loss (PL) in dB for a given path is [11]:

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$$PL(d)[dB] := \overline{PL}(d_0)[dB] + 10\alpha \log\left(\frac{d}{d_0}\right) + \tilde{X};\ \ d \geq d_0 \quad (1)$$

where $\overline{PL}(d_0)$ is average PL in dB at a reference distance d_0 from the transmitter, α is the path-loss exponent which depends on the propagation medium, and $\tilde{X}$ is a zero-mean Gaussian distributed random variable. In Dynamical model, the average PL becomes a random process denoted $\{X(t,\tau)\}_{t\geq 0, \tau\geq\tau_0}$, which is a function of both time t and location represented by τ, where $\tau = d/c$, d is the path length, c is the speed of light, $\tau_o = d_o/c$ and d_o is the reference distance. The process $\{X(t,\tau)\}_{t\geq 0, \tau\geq\tau_0}$ represents how much power the signal loses at a particular distance as a function of time. The signal attenuation is defined by $S(t,\tau) \triangleq e^{kX(t,\tau)}$, where $k = -c/2$ and $c = \ln(10)/10$. The process $X(t,\tau)$ is generated by a mean-reverting version of a general linear SDE given by [12]:

$$dX(t,\tau) = \beta(t,\tau)\big((\gamma(t,\tau) - X(t,\tau)\big)dt + \delta(t,\tau)dW(t),$$
$$X(t_0,\tau) = N(\overline{PL}(d)[dB];\sigma_{t_0}^2) \quad (2)$$

where $\{W(t)\}_{t\geq 0}$ is the standard Brownian motion (zero drift, unit variance) which is assumed to be independent of $X(t,\tau)$, and $N(\mu;\kappa)$ denotes a Gaussian random variable with mean μ and variance κ, and $\overline{PL}(d)[dB]$ is the average PL in dB. The parameter $\gamma(t,\tau)$ models the average time-varying PL at distance d from transmitter, which corresponds to $\overline{PL}(d)[dB]$ at d indexed by t. This model tracks and converges to this value as time progresses. The instantaneous drift $\beta(t,\tau)\big(\gamma(t,\tau) - X(t,\tau)\big)$ represents the effect of pulling the process towards $\gamma(t,\tau)$, while $\beta(t,\tau)$ represents the speed of adjustment towards this value. Finally, $\delta(t,\tau)$ controls the instantaneous variance or volatility of the process for the instantaneous drift.

Let $\{\theta(t,\tau)\}_{t\geq 0} \triangleq \{\beta(t,\tau), \gamma(t,\tau), \delta(t,\tau)\}_{t\geq 0}$. If the random processes $\{\theta(t,\tau)\}_{t\geq 0}$ are measurable and bounded, then (2) has a unique solution for every $X(t_0,\tau)$ as [12]:

$$X(t,\tau) = e^{-\beta([t,t_0],\tau)}\left(\begin{array}{l} X(t_0,\tau) + \int_{t_0}^{t} e^{\beta([u,t_0],\tau)}[\beta(u,\tau)\gamma(u,\tau)du \\ +\delta(u,\tau)dW(u)] \end{array}\right) \quad (3)$$

where $\beta([t,t_0],\tau) \triangleq \int_{t_0}^{t} \beta(u,\tau)du$. This model captures the temporal and spatial variations of the propagation environment as the random parameters $\{\theta(t,\tau)\}_{t\geq 0}$ model the time and space varying characteristics of the channel.

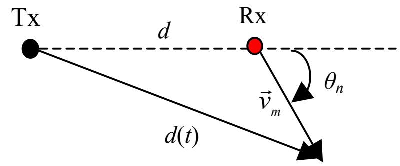


Fig. 1. A receiver moves at velocity υ_m, direction θ_n, and at a distance d from a transmitter.

It is required that the mean of PL process $E[X(t,\tau)]$ should track time and space variations of the average PL. The parameters $\beta(t,\tau)$ and $\delta(t,\tau)$ can be determined from experimental measurements.

Now consider a receiver at a distance d from a transmitter that moves at a constant velocity υ_m in a direction defined by an arbitrary constant angle θ_n, where θ_n is the angle between the direction of motion of the mobile and the distance vector that starts from the transmitter towards the receiver as shown in Figure 1. At time t, the distance from the transmitter to the receiver $d(t)$ is given by:

$$d(t) = \sqrt{d^2 + (\upsilon_m t)^2 + 2dt\upsilon_m \cos\theta_n} \quad (4)$$

Therefore, the average PL at that new location is [11]:

$$\gamma(t,\tau) = \overline{PL}(d(t))[dB] = \overline{PL}(d_0)[dB] + 10\alpha \log\frac{d(t)}{d_0} \quad (5)$$

where $d(t) \geq d_0$ and $\overline{PL}(d_0)$ is the average PL in dB at reference distance d_0, $d(t)$ is defined in (4), α is the power loss coefficient. This model is used to generate the link gains of the proposed PCA for LTF communication networks.

B. Short Term Fading (STF) Channel Model

The traditional STF model is based on Ossanna [13], and later expanded by Clarke [14], and Aulin, [15]. Aulin's model is shown in Figure 2. This model assumes that at each point between a transmitter and a receiver, the total received wave consists of superposition of N plane waves each having traveled via a different path.

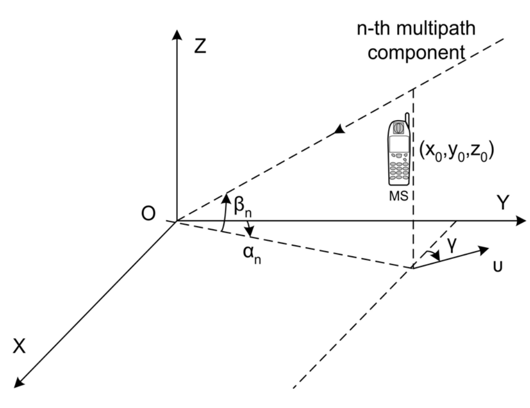


Fig. 2. Aulin's scattering model.

The nth wave is characterized by its field vector $E_n(t)$ as:

$$E_n(t) = I_n(t)\cos\omega_c t - Q_n(t)\sin\omega_c t = \mathrm{Re}\{r_n e^{j\Phi_n(t)} e^{j\omega_c t}\} \quad (6)$$

where $\{I_n(t), Q_n(t)\}$ are the corresponding inphase and quadrature components, $r_n(t) = \sqrt{I_n^2(t) + Q_n^2(t)}$ is the signal envelope, $\Phi_n(t) = \tan^{-1}(Q_n(t)/I_n(t))$ is the phase and ω_c is the carrier frequency and {Re} denotes the real part. The total field $E(t)$ is given by:

$$E(t) = I(t)\cos\omega_c t - Q(t)\sin\omega_c t \quad (7)$$

where $\{I(t), Q(t)\}$ are inphase and quadrature components of the total wave with $I(t) = \sum_{n=1}^{N} I_n(t)$ and $Q(t) = \sum_{n=1}^{N} Q_n(t)$.

The main idea in constructing the dynamical model for flat STF channels is to factorize the Doppler power spectral density (DPSD) into an approximate 4th order even transfer function, and then any stochastic realization can be used to obtain a state space representation for inphase and quadrature components. Consider the expression for the DPSD given by [15]:

$$S_D(f) = \begin{cases} 0, & |f| > f_m \\ \dfrac{E_0}{4 f_m \sin\beta_m}, & f_m\cos\beta_m \le |f| \le f_m \\ \dfrac{E_0}{4\pi f_m \sin\beta_m}\left[\dfrac{\pi}{2} - \sin^{-1}\left(\dfrac{2\cos^2\beta_m - 1 - (f/f_m)^2}{1-(f/f_m)^2}\right)\right], & |f| < f_m\cos\beta_m \end{cases} \quad (8)$$

where f_m is the maximum Doppler frequency, $E_0/2 = Var(I(t)) = Var(Q(t))$, and $\{\alpha, \beta\}$ define the direction of the incident wave onto the receiver as illustrated in Figure 2.

In order to approximate the power spectral density in (8), a 4th order even function in the form $\tilde{S}_D(s) = H(s)H(-s)$ with factorization [16]:

$$\tilde{S}_D(s) = \frac{K^2}{s^4 + 2\omega_n^2(1-2\zeta^2)s^2 + \omega_n^4},\ H(s) = \frac{K}{s^2 + 2\zeta\omega_n + \omega_n^2} \quad (9)$$

where $\tilde{S}_D(s)$ is the approximation of $S_D(s)$. Equation (9) has three arbitrary parameters $\{\zeta, \omega_n, K\}$, which can be adjusted such that the approximate curve coincides with the actual curve at different points. In fact, if these parameters are chosen such that:

$$\zeta = \sqrt{\frac{1}{2}\left(1 - \sqrt{1 - \frac{S_D(0)}{S_D(j\omega_{\max})}}\right)},\ \omega_n = \frac{\omega_{\max}}{\sqrt{1-2\zeta^2}},\ K = \omega_n^2\sqrt{S_D(0)} \quad (10)$$

then the approximate density $\tilde{S}_D(s)$ coincides with the exact density $S_D(s)$ at $\omega = 0$ and $\omega = \omega_{\max}$. The SDE, which corresponds to $H(s)$ in (9) is:

$$\ddot{x}(t) + 2\zeta\omega_n\dot{x} + \omega_n^2 x(t) = K\dot{w}(t),\quad \dot{x}(0), x(0) \text{ are given} \quad (11)$$

where $\{\dot{w}(t)\}_{t\ge 0}$ is a white-noise process. Equation (11) can be re-written in terms of inphase and quadrature components as:

$$\begin{aligned} &\ddot{x}_I(t) + 2\zeta\omega_n\dot{x}_I + \omega_n^2 x_I(t) = K\dot{w}_I(t),\ \dot{x}_I(0), x_I(0) \text{ are given} \\ &\ddot{x}_Q(t) + 2\zeta\omega_n\dot{x}_Q + \omega_n^2 x_Q(t) = K\dot{w}_Q(t),\ \dot{x}_Q(0), x_Q(0) \text{ are given} \end{aligned} \quad (12)$$

where $\{\dot{w}_I(t)\}_{t\ge 0}$ and $\{\dot{w}_Q(t)\}_{t\ge 0}$ are two independent and identically distributed (i.i.d) white Gaussian noises with distribution $\mathrm{N}(0;\sigma_\omega^2)$. Equations (12) can be realized in state-space controllable canonical form as:

$$\begin{aligned} &\dot{X}_I(t) = A_I X_I(t) + B_I\dot{w}_I(t), \quad X_I(0) \in \Re^2; \\ &\dot{X}_Q(t) = A_Q X_Q(t) + B_Q\dot{w}_Q(t), \quad X_Q(0) \in \Re^2, \end{aligned} \quad (13)$$

where $A_I = A_Q = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix}, B_I = B_Q = \begin{bmatrix} 0 \\ K \end{bmatrix}$, $X(t,\tau)$ is the PL, and $\Re^2$ denotes the real two dimensional Euclidean space. Define,

$$\begin{aligned} &A := \begin{bmatrix} A_I & 0 \\ 0 & A_Q \end{bmatrix},\ B := \begin{bmatrix} B_I & 0 \\ 0 & B_Q \end{bmatrix}, \\ &X := \begin{bmatrix} X_I \\ X_Q \end{bmatrix},\ \dot{w} := \begin{bmatrix} \dot{w}_I \\ \dot{w}_Q \end{bmatrix}, \upsilon := \begin{bmatrix} \upsilon_I \\ \upsilon_Q \end{bmatrix}, \\ &C(t) := [\cos\omega_c t \quad 0 \quad -\sin\omega_c t \quad 0], \\ &D(t) := [\cos\omega_c t \quad -\sin\omega_c t] \end{aligned} \quad (14)$$

then the stochastic state space realizations for Rayleigh and Rician flat fading channels can be described as [16]:

$$\begin{aligned} &\dot{X}(t) = AX(t) + f_s(t) + B\dot{w}(t), \quad X(0) \in \Re^4; \\ &y(t) = s(t)C(t)X(t) + D(t)\upsilon(t) \end{aligned} \quad (15)$$

where $\{y(t)\}_{t\ge 0}$ is the output signal and $\{\upsilon_I(t)\}_{t\ge 0}$, $\{\upsilon_Q(t)\}_{t\ge 0}$ are two i.i.d white Gaussian noises with distribution $\mathrm{N}(0;\sigma_\upsilon^2)$. And $f_s(t)$ describes the specular or LOS component. This model is used to generate the link gains for the proposed power control algorithm for STF communication networks.

III. POWER CONTROL MODEL

Consider a wireless network of M transmitters and M receivers. The measure of quality of service QoS can be defined by the SIR as [10]:

$$\min_{(p_1\ge 0,\ldots,p_M\ge 0)} \sum_{i=1}^{M} p_i \quad \text{subject to} \quad \frac{p_n g_{nn}}{\sum_{j\ne n}^{M} p_j g_{nj} + \eta_n} \ge \bar{\gamma}_n \quad (16)$$

which is equivalent to

$$\min_{(p_1\ge 0,\ldots,p_M\ge 0)} \sum_{i=1}^{M} p_i \quad \text{subject to} \quad \frac{p_n g_{nn}}{\sum_{j=1}^{M} p_j g_{nj} + \eta_n} \ge \gamma_n \quad (17)$$

where $\gamma_n \triangleq \dfrac{\bar{\gamma}_n}{\bar{\gamma}_n + 1}, 0 < \gamma_n < 1$. Here p_n denotes the power of transmitter n, $g_{nj} > 0$ denotes the channel gain of transmitter j to the receiver assigned to transmitter n, $\bar{\gamma}_n > 0$ is the required SIR threshold and $\eta_n > 0$ is the noise power level at the nth receiver, $1 \le n, j \le M$. The constraint in Equation (17) in the dynamic case for LTF channel using the path-wise QoS of each user with respect to the power signals over a time interval [0,*T*] is [17]:

$$\min_{(p_1\geq 0,\ldots,p_M\geq 0)} \left\{ \sum_{i=1}^{M} \int_0^T p_i(t) \right\}, \text{ subject to}$$

$$\frac{\int_0^T p_n(t)\left\|S_{nn}(t)\right\|^2 dt}{\sum_{j=1}^{M} \int_0^T p_j(t)\left\|S_{nj}(t)\right\|^2 dt + \int_0^T \left\|d_n(t)\right\|^2 dt} \geq \gamma_n \quad (18)$$

and for STF the constraint in (17) becomes [18]:

$$\frac{\int_0^T p_n(t)\left\|H_{nn}X_{nn}(t)\right\|^2 dt}{\sum_{j=1}^{M} \int_0^T p_j(t)\left\|H_{nj}(t)X_{nj}(t)\right\|^2 dt + \int_0^T \left\|d_n(t)\right\|^2 dt} \geq \gamma_n \quad (19)$$

where $S_{nj}(t)$ is the signal attenuation coefficients from transmitter *j* to receiver assigned to transmitter n at time t, $d_{nj}(t)$ is the channel disturbance at the *n*th receiver at time t, H_{nj}, $X_{nj}(t)$, $p_j(t)$, $d_n(t)$ are the same as defined in the dynamical STF channel model, $\|.\|$ is the Euclidean norm, and $n=1,\cdots,M$. The signal attenuation coefficients $S_{nj}(t)$ for LTF in (18) are generated using the SDE in (2) and the relation $S(t,\tau)=e^{kX(t,\tau)}$. The path losses $X_{nj}(t)$ for STF in (19) are generated using the SDE in (15).

PPCS is used to find the solution to (18) and (19). In wireless cellular networks, it is practical to observe and estimate channels at base stations and then communicate the information to the transmitters to adjust their control input signals $\{u_k(t)\}_{k=1}^{M}$. Since channel experiences delays, and the control are not feasible continuously in time but only at discrete time instants, the concept of predictable strategies is introduced [10]. Let the channel information at any time t be denoted by $\{I(t),Q(t),s(t)\}$ for STF and $\{S(t),s(t)\}$ for LTF. Let the control input signal for a transmitter at discrete time be $\{u(t);t_1,t_2,\cdots,T\}$. At time t_{j-1}, the base station observes the channel information $\{I_k(t_{j-1}),Q_k(t_{j-1}),s_k(t_{j-1})\}_{k=1}^{M}$ for STF or $\{S_k(t_{j-1}),s_k(t_{j-1})\}_{k=1}^{M}$ for LTF. Using the concept of predictable strategy, the base station determines the control strategy $\{u_k(t_j)\}_{k=1}^{M}$ for the next time instant t_j. The latter is communicated back to the transmitters that hold these values during the time interval $[t_{j-1},t_j)$. At time t_j, a new set of channel information $\{I_k(t_j),Q_k(t_j),s_k(t_j)\}_{k=1}^{M}$ or $\{S_k(t_j),s_k(t_j)\}_{k=1}^{M}$ is observed at the base station and the time t_{j+1} control strategies $\{u_k(t_{j+1})\}_{k=1}^{M}$ are computed and then communicated to the transmitters and held constant during the time interval $[t_j,t_{j+1})$. Such decision strategies are called predictable strategies. Using the concept of PPCS over any time interval defined as $[t_k,\, t_{k+1}]$, identities (18) and (19) are equivalent to [10]:

$$\min_{p_i(t_{k+1})} \sum_{i=1}^{M} p_i(t_{k+1}) \text{ subject to}$$
$$p(t_{k+1}) \geq \Gamma G_I^{-1}(t_k,t_{k+1}) \times (G(t_k,t_{k+1})p(t_{k+1}) + \eta(t_{k+1})) \quad (20)$$

where

$$g_{ni}(t_k,t_{k+1}) := \int_{t_k}^{t_{k+1}} \left\|H_{ni}(t)X_{ni}(t)\right\|^2 dt, \quad 1\leq n,i\leq M \quad \text{for STF}$$
$$:= \int_{t_k}^{t_{k+1}} \left\|S_{ni}(t)\right\|^2 dt, \quad 1\leq n,i\leq M \quad \text{for LTF}$$
$$\eta_{ni}(t_k,t_{k+1}) := \int_{t_k}^{t_{k+1}} \left\|d_n(t)\right\|^2 dt,\ 1\leq n,i\leq M$$
$$G_I(t_k,t_{k+1}) = diag(g_{11}(t_k,t_{k+1}),\cdots,g_{MM}(t_k,t_{k+1}))$$
$$G(t_k,t_{k+1}) = \{(g_{ni}(t_k,t_{k+1})\}_{M\times M},\ 1\leq n,i\leq M$$
$$\eta(t_k,t_{k+1}) = (\eta_1(t_k,t_{k+1}),\cdots,\eta_M(t_k,t_{k+1}))^T$$
$$p(t_{k+1}) = (p_1(t_{k+1}),\cdots,p_M(t_{k+1}))^T$$
$$\Gamma = diag(\gamma_1,\cdots,\gamma_M)$$

and Γ is the minimum required SIR for efficient communications, $diag(\cdot)$ denotes a diagonal matrix with its argument as diagonal entries, "*T*" stands for matrix or vector transpose. The optimization in (20) is a linear programming problem in $M\times 1$ vector of unknowns $p(t_k+1)$. Here $[t_k,\, t_{k+1}]$ is a time interval such that the channel model does not change significantly, i.e., shorter than the coherence time of the channel.

The performance measure is interference or outage probability. It is defined as the probability that a randomly chosen link will fail due to excessive interference [2]. Therefore, smaller outage probability implies larger capacity of the wireless network. A link with received SIR $\overline{\gamma}_{rcvd}$ less than or equal to threshold SIR $\overline{\gamma}_{th}$ is considered a communication failure. The outage probability $F(\overline{\gamma}_{rcvd})$ is expressed as $F(\overline{\gamma}_{th})$ = $\Pr\{\overline{\gamma}_{rcvd}\leq\overline{\gamma}_{th}\}$, where $F(\overline{\gamma}_{rcvd})$ is the distribution of $\overline{\gamma}_{rcvd}$.

Since power control only occurs at discrete time instants using PPCS, the iterative algorithm described in [4] and [5] can be used to determine the optimal transmitted powers as:

$$P_i^{n+1}(t_{k+1}) = \frac{\overline{\gamma}_i}{\overline{\gamma}_{rcvd}^{n}(t_k,t_{k+1})} P_i^n(t_{k+1}) \quad (21)$$

where $i = 1, \cdots, M$, and *n* is the number of iterations. It is shown in [4] and [5] that the iterative power control in (21) converges to the optimal (minimal) power vector. The numerical implementation of the iterative scheme can be carried out during processing in the intervals $[t_k,\, t_{k+1}]$.

To illustrate the efficiency of the proposed PCA, two numerical examples for LTF and STF are presented in the next section.

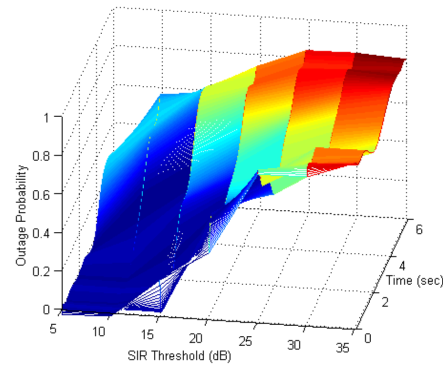
IV. NUMERICAL RESULTS

A. LTF Example

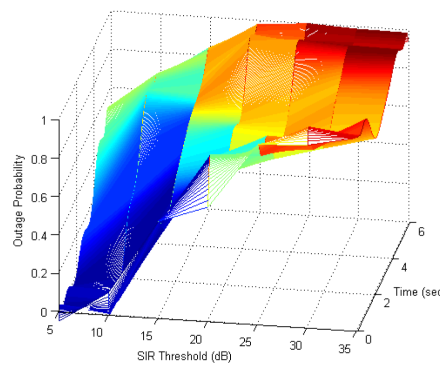
In this example, the performance of the proposed PCA for dynamical LTF channel is compared with the performance of no power control (NPC). The LTF cellular model has the following features:

- Number of transmitters and receivers is $M = 24$.
- Initial distances of all mobiles with respect to their own base stations d_{ii} are generated as uniformly independent identically distributed (*i.i.d.*) random variables (*r.v.*'s) in [10 – 100] meters.
- Cross initial distances of all mobiles with respect to other base stations $d_{ij}, i \neq j$, are generated as uniformly *i.i.d. r.v.*'s in [250 - 550] meters.
- The angle θ_{ij} between the direction of motion of mobile j and the distance vector passes through base station i and the mobile j are generated as uniformly *i.i.d. r.v.*'s in [0 – 180] degrees.
- The average velocities of mobiles are generated as uniformly *i.i.d. r.v.*'s in [40 – 100] km/hr.
- PL exponent is 3.5.
- Initial reference distance from each of the transmitters is 10 m.
- PL at the initial reference distance is 67 dB.
- $\delta(t) = 1400$ and $\beta(t) = 225000$ for the SDE's.
- η_n's are independent random variables with zero mean and variance = $4*10^{-8}$.

The SIR threshold $\bar{\gamma}_{th}$ is varied from five to thirty five in steps of five, and for each value of $\bar{\gamma}_{th}$ the outage probability is computed every 15 millisecond for 5 seconds. The outage probability is computed using Monte-Carlo simulations. The outage probability graphs of this example for both PC and NPC cases are shown in Figure 3a and 3b respectively. Figure 3 shows how the outage probability changes with respect to SIR threshold and time. As the SIR threshold increases the outage probability increase. This is obvious since we expect more users to fail. The outage probability is also changing with respect to time. This is because the mobiles are moving in different directions with different velocities. The average outage probability over all time intervals is shown in Figure 4. The outage probability is plotted versus SIR, which varies from 5 to 20 dB. The performance of PPCS is compared with the one of fixed transmit power or NPC. From Figure 4, it can be shown that the PPCS algorithm outperforms the reference algorithm by an order of magnitude. At outage probability below 0.2, SIR gains in excess of 10 dB may be achieved.



(a)



(b)

Fig. 3. Outage probability for dynamical long term fading model. (a) Using PPCS algorithm. (b) Using constant power transmit.

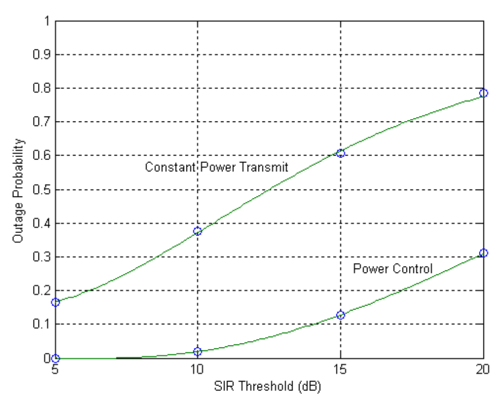


Fig. 4. Average outage probability for dynamical long term fading model. Performance comparison.

B. STF Example

In this example, the outage probability of the PPCS algorithm for both Rayleigh and Ricean fading channel models is calculated. The STF wireless cellular model has the following features:

- Number of transmitters/receivers, average velocities of mobiles, and η_n's are the same as in the LTF example.
- Carrier frequency = 910 MHz.
- E_{0ii}'s are independent random variables uniformly distributed in the range [400-600].

- E_{0ij}'s ($i \neq j$) are independent random variables uniformly distributed in the range [25-150].
- Angles of arrival $\beta_{m_{ij}}$'s for each link are generated as independent random variables uniformly distributed in [0 – 36] degrees, where $\beta_{m_{ij}}$ is the direction of the incident wave between transmitter j and receiver i.
- The parameters $\{\zeta, \omega_n, K\}$ are extracted from the DPSD as described in (10).

The average outage probabilities over all time intervals for both flat Rayleigh and flat Ricean are shown in Figure 5. The performance of PPCS is compared with the performance of fixed transmitter power. From Figure 5, it can be seen that the PPCS algorithm outperforms the reference algorithm. It is noticed that the outage probability for PPCS is less than the one for constant power transmit by at least 20%. It is also noticed that the performance of Ricean fading is better than the one of Rayleigh fading because of the LOS component in Ricean channels.

V. CONCLUSIONS

PPCS power control scheme is applied to dynamical long-term and short-term wireless communication channel models. More realistic time-varying wireless channel models are used than the usual static models encountered in the literature. The dynamics of the channel is depicted by SDE's, which essentially capture the spatio-temporal variations of wireless fading communication networks. Linear programming solves the optimization problem. Also iterative algorithms can be used to solve for the optimization problem. Numerical results presented for these algorithms indicate that the performance of PPCS outperforms the performance of constant power transmit. PPCS algorithm can be used as long as the channel model does not change significantly, that is the length of the intervals $[t_k, t_{k+1}]$ has to be shorter than the coherence time of the channel.

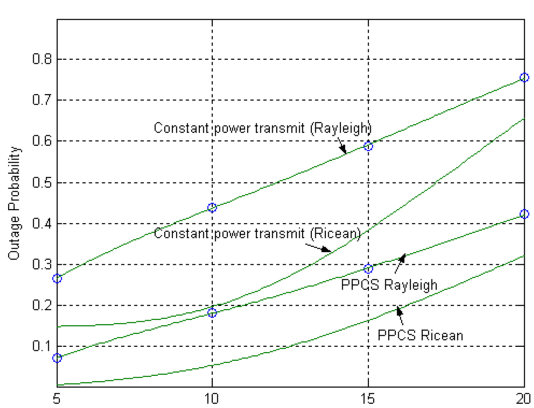


Fig. 5. Average outage probability for dynamical flat Rayleigh and Ricean short term fading model. Performance comparison.

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