Estimation and Identification of Time-Varying Long-Term Fading Wireless Channels with Application to Power Control

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Abstract—**This paper is concerned with modeling of time-varying wireless fading channels, parameter estimation, identification, and optimal power control from received signal measurements. Wireless channels are represented by stochastic differential equations, which parameters and state variables are estimated using Expectation Maximization and Kalman filtering, respectively. The latter are carried out solely from received signal measurements. Numerical results are presented to test the efficiency of the proposed channel estimation and identification algorithms. An optimal power control algorithm based on the estimated parameters and channel states is proposed. Numerical results indicate that a significant gain in performance can be achieved using the proposed approach.**

I. Introduction

Time-varying (TV) wireless channel models capture both the space and time variations of wireless systems, which are due to the relative mobility of the receiver and/or transmitter and scatterers [1]-[3]. This contrasts with the majority of published work that mainly deals with static random models or simple free space model [4]-[11]. This paper is concerned with the development of TV long-term fading (LTF) wireless channel models based on system identification and estimation algorithms to extract various parameters of the LTF channel using received signal measurements. The majority of research papers in this field such as in [4]-[6] use time-invariant (static) models for wireless channels. In time-invariant models, channel parameters are random but do not depend on time, and remain constant throughout the observation and estimation phase. This contrasts with TV models, where the channel dynamics become TV random (stochastic) processes [1]-[3].

The TV LTF channel model is introduced in [2], [3] and represented by stochastic differential equations (SDEs). We propose to estimate the TV power path-loss of the LTF channel and its parameters from received signal measurements, which are usually available or easy to obtain in any wireless network. The Expectation Maximization (EM) algorithm and Kalman filtering are employed in the identification and estimation processes. Numerical results are provided to determine the performance of the proposed estimation algorithm.

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The developed TV LTF channel model from received signal level measurements is useful in most wireless applications. It is used in developing an optimal power control algorithm (PCA), which is based on the estimated channel parameters from received signal measurements. The benefits of power minimization are not only increased battery life, but also increased overall network capacity. The power allocation problem has been studied extensively as an eigenvalue problem for non-negative matrices [7], [8], resulting in iterative PCAs that converge each user's power to the minimum power [9], [10], and as optimization-based approaches [11]. Much of this previous work deals with static time-invariant channel models.

The proposed PCA is based on predictable power control strategies (PPCS) that were first introduced in [1]. PPCS simply means updating the transmitted powers at discrete times and maintaining them fixed until the next power update begins. The PPCS algorithm is proven to be effectively applicable to such dynamical models for an optimal power control (PC). A distributed version of this algorithm is derived along the lines of [9] and [10], albeit based on the estimated model. The latter helps in allowing autonomous execution at the node or link level, requiring minimal usage of network resources for control signaling.

The paper is organized as follows. In Section II, the TV LTF mathematical channel model is introduced. In Section III, the EM algorithm together with the Kalman filter, to estimate the channel parameters as well as the channel power path-loss from received signal measurements, is developed. In Section IV, a PCA based on the proposed LTF channel model and the estimation algorithms is discussed. In Section V, numerical results are presented. Finally, Section VI provides the conclusion.

II. TV LTF Wireless Channel Mathematical Model

Wireless channels suffer from short-term fading (STF) due to multipath, and LTF due to shadowing depending on geographical area. In suburban areas, which are populated with less obstruction like vehicles, buildings, mountains and so forth, its communication signal undergoes phenomenal LTF (lognormal shadowing) [6].

The *time-invariant* power loss (PL) in dB is given by [6]:

$$PL(d)[\text{dB}] := \overline{PL}(d_0)[\text{dB}] + 10\alpha \log\left(\frac{d}{d_0}\right) + \tilde{Z}, \quad d \geq d_0 \qquad (1)$$

where $\overline{PL}(d_0)$ is the average PL in dB at a reference distance d_0 from the transmitter, the distance d corresponds to the transmitter-receiver separation distance, α is the path-loss exponent which depends on the propagating medium, and $\tilde{Z}$ is a zero-mean Gaussian random variable.

In TV environment, the random PL in (1) is relaxed to become a random process, denoted by $\{X(t,\tau)\}_{t\geq 0,\tau\geq\tau_0}$, which can be generated by a mean-reverting version of a general linear SDE given by [3]:

$$dX(t,\tau) = \beta(t,\tau)\big((\gamma(t,\tau) - X(t,\tau)\big)dt + \delta(t,\tau)dW(t),$$
$$X(t_0,\tau) = N(\overline{PL}(d)[dB];\sigma_{t_0}^2) \tag{2}$$

where $\{W(t)\}_{t\geq 0}$ is the standard Brownian motion (zero drift, unit variance) which is assumed to be independent of $X(t_0,\tau)$, $N(\mu;\kappa)$ denotes a Gaussian random variable with mean μ and variance κ, and $\overline{PL}(d)[dB]$ is the average PL in dB. The parameter $\gamma(t,\tau)$ models the average time-varying PL at distance d from transmitter, which corresponds to $\overline{PL}(d)[dB]$ at d indexed by t. This model tracks and converges to this value as time progresses. The instantaneous drift $\beta(t,\tau)\big(\gamma(t,\tau) - X(t,\tau)\big)$ represents the effect of pulling the process towards $\gamma(t,\tau)$, while $\beta(t,\tau)$ represents the speed of adjustment towards this value. Finally, $\delta(t,\tau)$ controls the instantaneous variance or volatility of the process for the instantaneous drift.

This model captures the spatio-temporal variations of the propagation environment as the random parameters $\{\beta(t,\tau),\gamma(t,\tau),\delta(t,\tau)\}_{t\geq 0}$ can be used to model the TV characteristics of the LTF channel. The received signal, $y(t)$, at any time t can be expressed as:

$$y(t) = s(t)S(t) + v(t) \tag{3}$$

where $s(t)$ is the information signal, $v(t)$ is the channel disturbance or noise at the receiver, and $S(t)$ is the signal attenuation coefficient defined by $S(t) \triangleq e^{kX(t,\tau)}$, where $k = -\ln(10)/20$ [6].

The general spatio-temporal lognormal model in (2) and (3) can be realized by stochastic state space given by:

$$\dot{X}(t,\tau) = A(t,\tau)X(t,\tau) + B(t,\tau)w(t)$$
$$y(t) = s(t)e^{kX(t,\tau)} + v(t) \tag{4}$$

where $A(t,\tau) = -\beta(t,\tau)$, $B(t,\tau) = \big[\delta(t,\tau) \;\; \beta(t,\tau)\gamma(t,\tau)\big]$ and $w(t) = \big[dW(t) \;\; 1\big]^T$.

The above system parameters and state variable values can be estimated from received signal measurements, which are usually available or easy to obtain in any wireless network. The EM algorithm and Kalman filtering are employed in the system parameters and state estimation, respectively. These algorithms are introduced next.

III. LTF Wireless Channel Estimation Via the EM Algorithm and Kalman Filtering

This section describes the procedure employed to estimate the channel model parameters and states associated with the state space model in (4), based on the EM algorithm [12] together with Kalman filtering [13]. However, for simplicity we consider the discrete-time version of (4) given by:

$$x_{t+1} = A_t x_t + B_t w_t$$
$$y_t = s_t e^{kx_t} + D_t v_t \tag{5}$$

where $x_t \in \Re^n$ is a state vector, $y_t \in \Re^d$ is a measurement vector, $w_t \in \Re^m$ is a state noise, and $v_t \in \Re^d$ is a measurement noise. Note that the state space model is nonlinear since the output equation in (5) is nonlinear. We consider the general form of state space form since the estimation algorithm is derived for the general case. However, in (4), we have $n = d = 1$ and $m = 2$.

The unknown system parameters $\theta_t = \{A_t, B_t, D_t\}$ as well as the system states x_t are estimated through a finite set of received signal measurement data, $Y_N = \{y_1, y_2, ..., y_N\}$. The methodology proposed is recursive and based on the EM algorithm combined with the extended Kalman filter (EKF). The latter is used due to the nonlinear output equation.

A. Channel State Estimation: The EKF

The EKF approach is based on linearizing the nonlinear system model (5) around the previous estimate. It estimates the channel states x_t for given system parameter $\theta_t = \{A_t, B_t, D_t\}$ and measurements Y_t. It is described by the following equations [13]:

$$\hat{x}_{t|t} = A_t\hat{x}_{t-1|t-1} + P_{t|t}C_t^T D_t^{-2}\left(y_t - C_t A_t \hat{x}_{t-1|t-1}\right)$$
$$\hat{x}_{t|t-1} = A_t\hat{x}_{t-1|t-1}$$
$$C_t = s_t \left.\frac{d\left(e^{kx_{t|t}}\right)}{dx_{t|t}}\right|_{x_{t|t}=\hat{x}_{t-1|t-1}} = s_t k e^{kx_{t|t}}\Big|_{x_{t|t}=\hat{x}_{t-1|t-1}} \tag{6}$$

where $t = 0,1,2,...,N$, and $P_{t|t}$ is given by:

$$\overline{P}_{t|t}^{-1} = P_{t-1|t-1}^{-1} + A_t^T B_t^{-2} A_t$$
$$P_{t|t}^{-1} = C_t^T D_t^{-2} C_t + B_t^{-2} - B_t^{-2}\overline{P}_{t|t}A_t^T B_t^{-2} \tag{7}$$
$$P_{t|t-1} = A_t P_{t-1|t-1} A_t^T + B_t^2$$

The channel parameters $\theta_t = \{A_t, B_t, D_t\}$ are estimated based on the EM algorithm, which is introduced next.

B. Channel Parameter Estimation: The EM Algorithm

The EM algorithm uses a bank of Kalman filters to yield a maximum likelihood (ML) parameter estimate of the state space model. It is an iterative scheme for computing the ML estimate of the system parameters θ_t, given the data Y_t. Specifically, each iteration of the EM algorithm consists of two steps: The expectation and the maximization steps.

The expectation step evaluates the conditional expectation of the log-likelihood function given the complete data as:

$$\Lambda(\theta_t,\hat{\theta}_t)=E_{\theta_l}\left\{\log\frac{dP_{\theta_t}}{dP_{\hat{\theta}t}}\,|\,Y_t\right\} \tag{8}$$

where $\hat{\theta}_t$ denotes the estimated system parameters at time step *t*. The maximization step finds:

$$\hat{\theta}_{t+1}\in\underset{\theta_t\in\Theta}{\arg\max}\ \Lambda\left(\theta_t,\hat{\theta}_t\right) \tag{9}$$

The expectation and maximization steps are repeated until the sequence of model parameters converge to the real parameters. The EM algorithm is given by [12]:

$$\begin{aligned}
\hat{A}_t&=E\left(\sum_{k=1}^{t}x_kx_{k-1}^T\,|\,Y_t\right)\times\left[E\left(\sum_{k=1}^{t}x_kx_k^T\,|\,Y_t\right)\right]^{-1}\\
\hat{B}_t^2&=\frac{1}{t}E\left(\sum_{k=1}^{t}\left(\left(x_k-A_kx_{k-1}\right)\left(x_k-A_kx_{k-1}\right)^T\right)|\,Y_t\right)\\
&=\frac{1}{t}E\left(\sum_{k=1}^{t}\begin{pmatrix}\left(x_kx_k^T\right)-A_k\left(x_kx_{k-1}^T\right)^T\\-\left(x_kx_{k-1}^T\right)A_k^T+A_k\left(x_{k-1}x_{k-1}^T\right)A_k^T\end{pmatrix}|\,Y_t\right)\\
\hat{D}_t^2&=\frac{1}{t}E\left(\sum_{k=1}^{t}\left(\left(y_k-C_kx_k\right)\left(y_k-C_kx_k\right)^T\right)|\,Y_t\right)\\
&=\frac{1}{t}E\left(\sum_{k=1}^{t}\begin{pmatrix}\left(y_ky_k^T\right)-A_k\left(y_kx_k^T\right)C_k^T\\-C_k\left(y_kx_k^T\right)^T+C_k\left(x_kx_k^T\right)C_k^T\end{pmatrix}|\,Y_t\right)
\end{aligned} \tag{10}$$

where $B_t^2=B_tB_t^T$, $D_t^2=D_tD_t^T$, $E(\cdot)$ denotes the expectation operator, and $t=0,1,2,...,N$. These system parameters $\left\{\hat{A}_t,\hat{B}_t^2,\hat{D}_t^2\right\}$ can be computed from the following conditional expectations [12]:

$$\begin{aligned}
L_t^{(1)}&=E\left\{\sum_{k=1}^{t}x_k^TQx_k\,|\,Y_t\right\}\\
L_t^{(2)}&=E\left\{\sum_{k=1}^{t}x_{k-1}^TQx_{k-1}\,|\,Y_t\right\}\\
L_t^{(3)}&=E\left\{\sum_{k=1}^{t}\left[x_k^TRx_{k-1}+x_{k-1}^TR^Tx_k\right]|\,Y_t\right\}\\
L_t^{(4)}&=E\left\{\sum_{k=1}^{t}\left[x_k^TSy_k+y_k^TS^Tx_k\right]|\,Y_t\right\}
\end{aligned} \tag{11}$$

where *Q*, *R* and *S* are given by:

$$\begin{aligned}
&Q=\left\{\frac{e_ie_j^T+e_je_i^T}{2}\right\};\ R=\left\{\frac{e_ie_j^T}{2}\right\};\ i,j=1,2,...n\\
&S=\left\{\frac{e_ie_j^T}{2};\ i=1,2,...n;\ j=1,2,..d\right\}
\end{aligned} \tag{12}$$

in which e_i is the unit vector in the Euclidean space; that is $e_i=1$ in the *i*th position, and 0 elsewhere. For instance, consider the case *n* = *d* = 1, then $E\left(\sum_{k=1}^{t}x_kx_{k-1}^T\,|\,Y_t\right)$ is:

$$E\left(\sum_{k=1}^{t}x_kx_{k-1}^T\,|\,Y_t\right)=L_t^{(3)}\left(R=\frac{1}{2}\right) \tag{13}$$

The other terms in (10) can be computed similarly.

The conditional expectations $\left\{L_t^{(1)},L_t^{(2)},L_t^{(3)},L_t^{(4)}\right\}$ can be estimated from measurements Y_t as follows:

1) Filter estimate of $L_t^{(1)}$:

$$\begin{aligned}
L_t^{(1)}&=E\left\{\sum_{k=1}^{t}x_k^TQx_k\,|\,Y_t\right\}\\
&=-\frac{1}{2}Tr\left(N_t^{(1)}P_{t|t}\right)-\frac{1}{2}\sum_{k=1}^{t}Tr\left(N_{k-1}^{(1)}\bar{P}_{k|k}\right)\\
&-\frac{1}{2}\sum_{k=1}^{t}\begin{pmatrix}-2x_{k|k}^TP_{k|k}^{-1}r_k^{(1)}+2x_{k|k-1}^TP_{k|k-1}^{-1}r_{k|k-1}^{(1)}-x_{k|k}^TN_k^{(1)}x_{k|k}\\+x_{k|k-1}^TB_k^{-2}A_k\bar{P}_{k|k}N_{k-1}^{(1)}\bar{P}_{k|k}A_k^TB_k^{-2}x_{k|k-1}\end{pmatrix}
\end{aligned} \tag{14}$$

where $Tr(\cdot)$ denotes the matrix trace. In (14), $r_k^{(1)}$ and $N_k^{(1)}$ satisfy the following recursions:

$$\begin{aligned}
&\begin{cases}r_k^{(1)}=\left(A_k-P_{k|k}C_k^TD_k^{-2}C_kA_k\right)r_{k-1}^{(1)}+2P_{k|k}Qx_{k|k-1}\\
\quad-P_{k|k}N_k^{(1)}P_{k|k}C_k^TD_k^{-2}\left(y_k-C_kx_{k|k-1}\right)\\
r_{k|k-1}^{(1)}=A_kr_k^{(1)}\\
r_0^{(1)}=0_{m\times1}\end{cases}\\
&\begin{cases}N_k^{(1)}=B_k^{-2}A_k\bar{P}_{k|k}N_{k-1}^{(1)}\bar{P}_{k|k}A_k^TB_k^{-2}-2Q\\
N_0^{(1)}=0_{m\times m}\end{cases}
\end{aligned} \tag{15}$$

2) Filter estimate of $L_t^{(2)}$:

$$\begin{aligned}
L_t^{(2)}&=E\left\{\sum_{k=1}^{t}x_{k-1}^TQx_{k-1}\,|\,Y_t\right\}\\
&=E_\theta\left\{x_0^TQx_0\,|\,Y_t\right\}+E_\theta\left\{\sum_{k=1}^{t}x_k^TQx_k\,|\,Y_t\right\}-E_\theta\left\{x_t^TQx_t\,|\,Y_t\right\}
\end{aligned} \tag{16}$$

Therefore, $L_t^{(2)}$ can be obtained from $L_t^{(1)}$.

3) Filter estimate of $L_t^{(3)}$:

$$\begin{aligned}
L_t^{(3)}&=E\left\{\sum_{k=1}^{t}\left(x_k^TRx_{k-1}+x_{k-1}^TR^Tx_k\right)|\,Y_t\right\}\\
&=-\frac{1}{2}Tr\left(N_t^{(3)}P_{t|t}\right)-\frac{1}{2}\sum_{k=1}^{t}Tr\left(N_{k-1}^{(3)}\bar{P}_{k|k}\right)\\
&-\frac{1}{2}\sum_{k=1}^{t}\begin{pmatrix}-2x_{k|k}^TP_{k|k}^{-1}r_k^{(3)}+2x_{k|k-1}^TP_{k|k-1}^{-1}r_{k|k-1}^{(3)}-x_{k|k}^TN_k^{(3)}x_{k|k}\\+x_{k|k-1}^TB_k^{-2}A_k\bar{P}_{k|k}N_{k-1}^{(3)}\bar{P}_{k|k}A_k^TB_k^{-2}x_{k|k-1}\end{pmatrix}
\end{aligned} \tag{17}$$

In this case, $r_k^{(3)}$ and $N_k^{(3)}$ satisfy the following recursions:

$$\begin{aligned}
&\begin{cases}r_k^{(3)}=\left(A_k-P_{k|k}C_k^TD_k^{-2}C_kA_k\right)r_{k-1}^{(3)}-P_{k|k}N_k^{(3)}P_{k|k}C_k^TD_k^{-2}\\
\quad\left(y_k-C_kx_{k|k-1}\right)+\left(2P_{k|k}R+2P_{k|k}B_k^{-2}A_k\bar{P}_{k|k}R^TA_k\right)x_{k-1|k-1}\\
r_{k|k-1}^{(3)}=A_kr_k^{(3)}\\
r_0^{(3)}=0_{m\times1}\end{cases}\\
&\begin{cases}N_k^{(3)}=B_k^{-2}A_k\bar{P}_{k|k}N_{k-1}^{(3)}\bar{P}_{k|k}A_k^TB_k^{-2}-2R\bar{P}_{k|k}A_k^TB_k^{-2}\\
\quad-2B_k^{-2}A_k\bar{P}_{k|k}R^T\\
N_0^{(3)}=0_{m\times m}\end{cases}
\end{aligned} \tag{18}$$

4) Filter estimate of $L_t^{(4)}$:

$$L_t^{(4)} = E\left\{\sum_{k=1}^{t}\left(x_k^T S y_k + y_k^T S^T x_k\right) \mid Y_t\right\} = \sum_{k=1}^{t}\left(x_{k|k}^T P_{k|k}^{-1} r_k^{(4)} - x_{k|k-1}^T P_{k|k-1}^{-1} r_{k|k-1}^{(4)}\right) \quad (19)$$

where $r_k^{(4)}$ satisfy the following recursions:

$$\begin{cases} r_k^{(4)} = (A_k - P_{k|k} C_k^T D_k^{-2} C_k A_k) r_{k-1}^{(4)} + 2 P_{k|k} S y_k \\ r_{k|k-1}^{(4)} = A_k r_k^{(4)} \\ r_0^{(4)} = 0_{m\times 1} \end{cases} \quad (20)$$

Using the filters for $L_t^{(i)}$ $(i = 1,2,3,4)$ and the extended Kalman filter described in (6) and (7), the system parameters $\theta_t = \{A_t, B_t, D_t\}$ are estimated through the EM algorithm described in (10). Numerical results that show the applicability of the above algorithm are discussed in Section V. In the next section, we introduce stochastic PCA based on the estimated channel models.

IV. Stochastic Power Control Algorithm in TV LTF Wireless Networks

The aim of the PCA described here is to minimize the total transmitted power of all users while maintaining acceptable QoS for each user. The measure of QoS is defined by the signal-to-interference ratio (SIR) for each link to be larger than a target SIR. Since the channel model parameters are estimated from received signal measurements, PC is performed only from these measurements.

Now consider a wireless network with *M* transmitters and *N* receivers. The state space representation of LTF wireless network can be written as:

$$\begin{aligned} \dot{X}_{ij}(t,\tau) &= A_{ij}(t,\tau) X_{ij}(t,\tau) + B_{ij}(t,\tau) w_{ij}(t) \\ y_i(t) &= \sum_{k=1}^{M} \sqrt{p_k(t)} s_k(t) e^{k X_{ik}(t,\tau)} + v_i(t) \end{aligned} \quad (21)$$

where $y_i(t)$ is the received signal at the *i*th receiver at time *t*, $X_{ij}(t)$ is the states of the TV PL of the channel between transmitter *j* and the receiver assigned to transmitter *i*, $p_k(t)$ is the transmitted power of transmitter *k* at time *t*, which acts as a scaling on the information signal $s_k(t)$, $v_i(t)$ is the channel disturbance or noise at receiver *i*, and $1 \le i, j \le M$.

Consider the wireless network described above, the centralized PC problem for *time-invariant* channels is [1]:

$$\min_{(p_1 \ge 0,\dots,p_M \ge 0)} \sum_{i=1}^{M} p_i \quad \text{subject to} \quad \frac{p_i g_{ii}}{\sum_{k\ne i}^{M} p_k g_{ik} + \eta_i} \ge \varepsilon_i, \quad 1 \le i \le M \quad (22)$$

where p_i is the power of transmitter *i*, $g_{ik} > 0$ is the *time-invariant* channel gain between transmitter *k* and the receiver assigned to transmitter *i*, $\varepsilon_i > 0$ is the target SIR of transmitter *i*, and $\eta_i > 0$ is the noise power level at the receiver *i*. Expression (22) for the TV LTF wireless network in (21), described using path-wise QoS of each user over a time interval [0,*T*] is given by:

$$\min_{(p_1 \ge 0,\dots,p_M \ge 0)} \left\{\sum_{i=1}^{M} \int_0^T p_i(t)\,dt\right\}, \quad \text{subject to} \quad \frac{\int_0^T p_i(t) s_i^2(t) S_{ii}^2(t)\,dt}{\sum_{k\ne i}^{M} \int_0^T p_k(t) s_k^2(t) S_{ik}^2(t)\,dt + \int_0^T v_i^2(t)\,dt} \ge \varepsilon_i \quad (23)$$

where $S_{ik}(t) = e^{k X_{ik}(t,\tau)}$ and $i = 1, \cdots, M$. A solution to (23) is presented by first introducing PPCS. Since channels experience delays, and power control is not feasible continuously in time but only at discrete-time instants, the concept of PPCS is introduced [1]. Consider a set of discrete time strategies $\{p_i(t_k)\}_{i=1}^M$, $0 = t_0 < t_1 < \ldots < t_k < t_{k+1} < \ldots \le T$. At time t_{k-1}, the base stations estimate the channel information $\{S_{ij}(t_{k-1}), s_i(t_{k-1})\}_{i,j=1}^M$ as described in Section III. The base stations then determine the control strategies $\{p_i(t_k)\}_{i=1}^M$ for the next time instant t_k. The latter is communicated back to the mobiles, which hold these values during the time interval $[t_{k-1}, t_k)$. At time t_k, a new set of channel information $\{S_{ij}(t_k), s_i(t_k)\}_{i,j=1}^M$ is estimated at the base stations and the time t_{k+1} control strategies $\{p_i(t_{k+1})\}_{i=1}^M$ are computed and communicated back to the mobiles which hold them constant during the time interval $[t_k, t_{k+1})$. Using the concept of PPCS over any time interval $[t_k, t_{k+1}]$, equation (23) is equivalent to:

$$\min_{\mathbf{p}(t_{k+1})>0} \sum_{i=1}^{M} p_i(t_{k+1}) \quad \text{subject to} \quad \mathbf{p}(t_{k+1}) \ge \mathbf{\Gamma}\mathbf{G}_I^{-1}(t_k, t_{k+1}) \times \left(\mathbf{G}(t_k, t_{k+1})\mathbf{p}(t_{k+1}) + \boldsymbol{\eta}(t_{k+1})\right) \quad (24)$$

where

$$g_{ij}(t_k, t_{k+1}) := \int_{t_k}^{t_{k+1}} s_j^2(t) S_{ij}^2(t)\,dt, \; 1 \le i, j \le M,$$

$$\mathbf{p}(t_{k+1}) := \left(p_1(t_{k+1}), \cdots, p_M(t_{k+1})\right)^T, \; \eta_i(t_k, t_{k+1}) := \int_{t_k}^{t_{k+1}} v_i^2(t)\,dt,$$

$$\mathbf{G}_I(t_k, t_{k+1}) := diag\left(g_{11}(t_k, t_{k+1}), \cdots, g_{MM}(t_k, t_{k+1})\right),$$

$$\mathbf{G}(t_k, t_{k+1}) := \begin{cases} 0 & \text{, if } i = j \\ g_{ij}(t_k, t_{k+1}) & \text{, if } i \ne j \end{cases}, \; 1 \le i, j \le M,$$

$$\boldsymbol{\eta}(t_k, t_{k+1}) := \left(\eta_1(t_k, t_{k+1}), \cdots, \eta_M(t_k, t_{k+1})\right)^T,$$

$\mathbf{\Gamma} := diag(\varepsilon_1, \cdots, \varepsilon_M)$, and $diag(\cdot)$ denotes a diagonal matrix with its argument as diagonal entries. The optimization in (24) is a linear programming problem in $M \times 1$ vector of unknowns $\mathbf{p}(t_{k+1})$. Throughout this section, we assume that the PC problem is feasible, i.e., there exists a power vector $\mathbf{p}(t_k)$ that satisfies the inequality in (24) for all $[t_k, t_{k+1}]$ in [0, T].

Next, we consider an iterative distributed version of the centralized PCA in (24). This is convenient for online implementation since it helps autonomous execution at the node or link level, requiring minimal usage of network communication resources for control signaling. The constraint in (24) can be written as:

$$\left(\mathbf{I}-\boldsymbol{\Gamma}\mathbf{G}_I^{-1}(t_k,t_{k+1})\mathbf{G}(t_k,t_{k+1})\right)\mathbf{p}(t_{k+1}) \geq \boldsymbol{\Gamma}\mathbf{G}_I^{-1}(t_k,t_{k+1})\boldsymbol{\eta}(t_{k+1}) \qquad (25)$$

Defining $\mathbf{F}(t_k,t_{k+1}) \triangleq \boldsymbol{\Gamma}\mathbf{G}_I^{-1}(t_k,t_{k+1})\mathbf{G}(t_k,t_{k+1})$ and $\mathbf{u}(t_k,t_{k+1}) \triangleq \boldsymbol{\Gamma}\,\mathbf{G}_I^{-1}(t_k,t_{k+1})\,\boldsymbol{\eta}(t_{k+1})$, then (25) can be written as:

$$\left(\mathbf{I}-\mathbf{F}(t_k,t_{k+1})\right)\mathbf{p}(t_{k+1}) \geq \mathbf{u}(t_k,t_{k+1}) \qquad (26)$$

If the channel gains are *time-invariant*, i.e., $\mathbf{F}(t_k,t_{k+1})=\mathbf{F}$ and $\mathbf{u}(t_k,t_{k+1})=\mathbf{u}$, then the PC problem is feasible if $\rho_{\mathbf{F}} < 1$, where $\rho_{\mathbf{F}}$ is the Perron-Frobenius eigenvalue of $\mathbf{F}$ [9]. It is shown in [9] and [10] that the following iterative PCA converges to the minimal power vector when $\rho_{\mathbf{F}} < 1$:

$$\mathbf{p}(t_{k+1})=\mathbf{F}\mathbf{p}(t_k)+\mathbf{u} \qquad (27)$$

However, our channel gains are *time-varying*, thus a time-varying version of the PCA in (27) can be defined as:

$$\mathbf{p}(t_{k+1})=\mathbf{F}(t_k,t_{k+1})\mathbf{p}(t_k)+\mathbf{u}(t_k,t_{k+1}) \qquad (28)$$

Clearly, in general the power vector $\mathbf{p}(t_k)$ will not converge to some deterministic constant as it does in (27). Since $\mathbf{F}(t_k,t_{k+1})$ is a random matrix-valued process, the key convergence condition is that the Lyapunov exponent $\lambda_{\mathbf{F}} < 0$ [14], where $\lambda_{\mathbf{F}}$ is defined as:

$$\lambda_{\mathbf{F}} = \lim_{k\to\infty}\frac{1}{k}\log\left\|\mathbf{F}(t_0,t_1)\mathbf{F}(t_1,t_2)\ldots\mathbf{F}(t_k,t_{k+1})\right\| \qquad (29)$$

The distributed version of (28) can be written as:

$$p_i(t_{k+1})=\frac{\varepsilon_i(t_k)}{R_i(t_k)}p_i(t_k),\ i=1,\ldots,M \qquad (30)$$

where $R_i(t_k)$ is the instantaneous SIR defined by:

$$R_i(t_k)=\frac{p_i(t_k)g_{ii}(t_k,t_{k+1})}{\sum_{j\neq i}^{M}p_j(t_k)g_{ij}(t_k,t_{k+1})+\eta_i(t_k,t_{k+1})} \qquad (31)$$

Note that the PCAs in (24) and (30) can be used as long as the channel model does not change significantly, that is $[t_k, t_{k+1}]$ is a subset of the coherence time of the channel.

In the next section, a numerical example is presented to determine the performance of the proposed PCA under the estimated LTF wireless channel models.

V. Numerical Examples

Two numerical examples are presented. In Example 1, the accuracy of the developed EM algorithm together with the extended Kalman filter to estimate channel parameters, as well as channel PL from the received signal measurements, is determined. In Example 2, we compare the performance of the proposed PCA using PPCS under TV LTF stochastic and static channel models.

Example 1:

The estimation of a LTF wireless channel from received signal measurements is considered. In particular, the estimation includes the channel parameters, channel PL, and received signal. The measurement data are generated by the following system parameters:

$$\gamma(t,\tau)=25\left(1+0.15e^{-2t/T}\sin\left(\frac{10\pi t}{T}\right)\right),\ \delta(t,\tau)=5,\ \beta(t,\tau)=0.2 \qquad (32)$$

and the variances of the state and measurement noises are 10^{-2} and 10^{-6}, respectively. Figure 1 shows the actual and estimated received signal using the EM algorithm together with the extended Kalman filter for 500 sampled data. From Figure 1, it can be noticed that the received signal have been estimated with very good accuracy. Figure 2 shows the received signal estimates root mean square error (RMSE) for 100 runs. It can be noticed that it takes just few iterations (less than 15) for the filter to converge, and the steady state performance of the proposed channel estimation algorithm using the EM together with Kalman filtering is excellent.

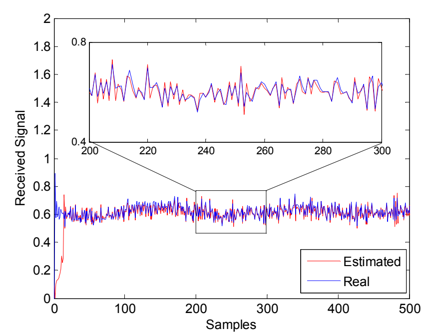


Fig. 1. Real and estimated received signal for the channel model in Ex. 1.

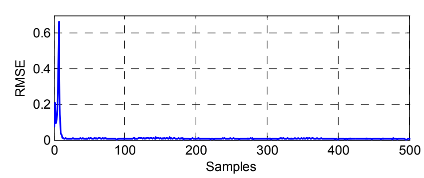
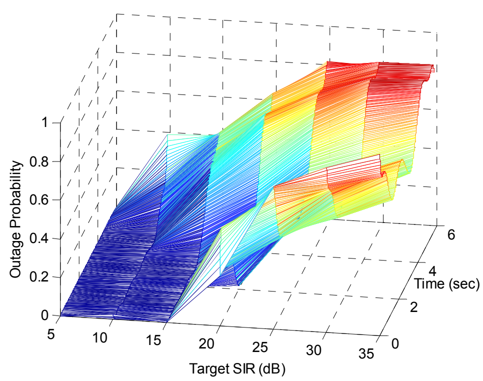


Fig. 2. Received signal estimates RMSE for 100 runs using the EM algorithm together with the extended Kalman filter.

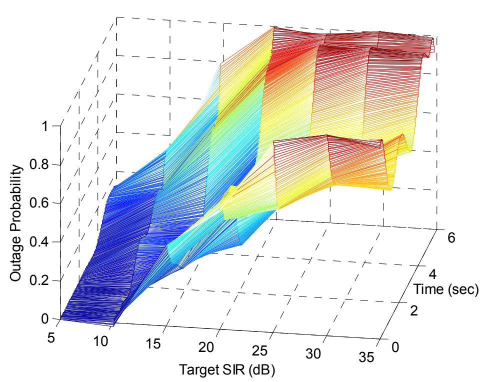
Example 2:

The cellular model setup in this example is the same as in [3] with M = 24 transmitters. The channel model parameters as well as the channel PL for all users are estimated online from received signal measurements using the EM algorithm together with the extended Kalman filter as illustrated in Example 1. The PCA described in (24) is performed using the estimated channel parameters and states. The outage probability (OP) is used as a performance measure for the PCA. A link with a received SIR R_i, less than or equal to a target SIR ε_i, is considered a communication failure. The OP, $O(\varepsilon_i)$, is expressed as $O(\varepsilon_i) = \text{Prob}\{R_i \leq \varepsilon_i\}$. The targets SIR, ε_i, for all users are the same, and varied from 5 dB to 35 dB with step 5 dB. For each value of ε_i the OP is computed every 15 millisecond, i.e., $[t_k, t_{k+1}] = 15$ millisecond. The simulation is performed for 6 seconds, i.e., $[0, T] = 6$ seconds. The OP is computed using Monte-Carlo simulations. In this example, we compare the performance of the proposed PCA using PPCS described in (24) under two different types of TV LTF channel models; the stochastic model in (2) and the static model in (1).

The OP for the PCA using PPCS based on both stochastic and static TV LTF channel models are shown in Figure 3a and 3b, respectively. Figure 3 shows how the OP changes with respect to the target SIR, ε_i, and time. The OP changes as a function of time, since mobiles move in different directions and velocities. The average OP versus ε_i over the whole simulation time (6 seconds) is shown in Figure 4. It can be noticed that the performance of PC based on PPCS using the stochastic models is on average much better than that of static models. This is because the static models do not capture the time-variations of the channels. For example, at 20 dB target SIR, the OP is reduced from 0.45 for static models to 0.3 for TV stochastic ones; this represents an improvement of over 33%.



(a)



(b)

Fig. 3. OP for the PCA using PPCS under TV LTF wireless networks for (a) stochastic channel models. (b) static channel models.

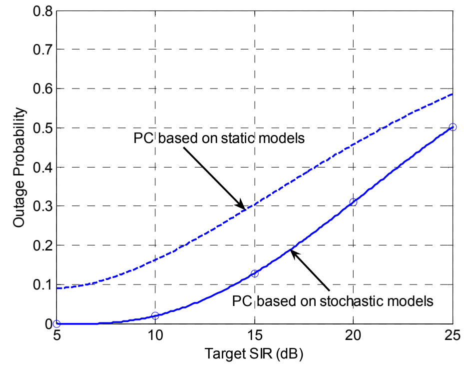


Fig. 4. Average OP for the PCA using PPCS under TV LTF wireless networks. Performance comparison.

VI. Conclusion

This paper describes a general scheme for extracting mathematical LTF channel models from noisy received signal measurements, and performing power control based on the estimated channel parameters. The proposed estimation algorithm consists of filtering based on the extended Kalman filter to remove noise from data, and identification based on the EM algorithm to determine the parameters of the model which best describe the measurements.

References

[1] C.D. Charalambous, S.M. Djouadi, and S.Z. Denic, "Stochastic power control for wireless networks via SDE's: Probabilistic QoS measures," *IEEE Trans. on Inform. Th.*, vol. 51, No. 2, pp. 4396-4401, Dec. 2005.

[2] M.M. Olama, S.M. Shajaat, S.M. Djouadi and C.D. Charalambous, "Stochastic power control for time-varying long term fading wireless channels," *Proceedings of the American Control Conference*, pp. 1817-1822, Portland, Oregon, USA, June 8-10, 2005.

[3] Mohammed M. Olama, Seddik M. Djouadi, and Charalambos D. Charalambous, "Stochastic power control for time-varying long-term fading wireless networks," *EURASIP Journal on Applied Signal Processing*, vol. 2006, Article ID 89864, 13 pages, 2006.

[4] W. Jakes, *Microwave Mobile Communications*, IEEE, Inc. NY, 1974.

[5] J. Proakis, *Digital Communications*, 4th Edition, McGraw Hill, 2000.

[6] T.S. Rappaport, *Wireless Communications: Principles and Practice*, Prentice Hall, 2nd Edition, 2002.

[7] J. Zander, "Performance of optimum transmitter power control in cellular radio systems," *IEEE Trans. on Veh. Tech.*, vol. 41, no.1, Feb. 1992.

[8] J. Aein, "Power balancing in systems employing frequency reuse," *COMSAT Technical Review*, vol. 3, 1973.

[9] N. Bambos and S. Kandukuri, "Power-controlled multiple access schemes for next-generation wireless packet networks," *IEEE Wireless Communications*, vol. 9, issue 3, June 2002.

[10] G.J. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. on Vehicular Tech.*, vol. 42, no.4, Nov. 1993.

[11] S. Kandukuri and S. Boyd, "Optimal power control in interference-limited fading wireless channels with outage-probability specifications," *IEEE Transactions on Wireless Communications*, vol. 1, no. 1, pp. 46-55, 2002.

[12] C.D. Charalambous and A. Logothetis, "Maximum-likelihood parameter estimation from incomplete data via the sensitivity equations: The continuous-time case," *IEEE Transaction on Automatic Control*, vol. 45, no. 5, pp. 928-934, May 2000.

[13] G. Bishop and G. Welch, *An introduction to the Kalman filters*, University of North Carolina, 2001.

[14] A.I. Mees, *Nonlinear Dynamics and Statistics*, Boston, 2001.