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Estimation of Mobile Station Position and Velocity in Multipath Wireless Networks Using the Unscented Particle Filter

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Abstract— **This paper presents a method based on wave scattering model for tracking a user. The 3D wave scattering multipath channel model of Aulin is employed together with particle filtering to obtain mobile station location and velocity estimates with high accuracy. This model takes into account non-line-of-sight and multipath propagation environments, which are usually encountered in wireless fading channels. The proposed estimation algorithms are based on the particle filter (PF) and the unscented particle filter (UPF). These algorithms cope with nonlinearities in the channel model in order to estimate the mobile location and velocity. They do not rely on linearized motion models, measurement relations, and Gaussian assumptions, in contrast to the extended Kalman filter (EKF). The performance of the PF/UPF approaches outperforms the EKF approach as simulation results indicate. Moreover, numerical results are presented to evaluate the performance of the proposed algorithms when measurement data do not correspond to the ones generated by the model. This shows the robustness of the algorithm.**

I. Introduction

The need for an efficient and accurate mobile station (MS) positioning system is growing day by day. This has been stressed by a federal order issued by the federal communications commission (FCC), which mandates all wireless service providers to provide public safety answering points with information to locate an emergency 911 caller with an accuracy of 100 meters for 67% of the cases [1]. It is also expected that the FCC will tighten its requirements in the near future [2]. Many other applications, such as vehicle fleet management, location sensitive billing, intelligent transport systems, and mobile yellow pages have driven the cellular industry to research new and promising technologies for MS positioning.

The problem of determining the location and velocity of a MS has been studied extensively in the last few years. The current standards in estimating the location and velocity are based mostly on time signal information [3]-[8]. However, not all of these methods meet the necessary needs imposed by certain services. In addition, most of them require new hardware since localization is not inherent in the current wireless systems. Researchers have also suggested several MS location methods based on signal power measurements such as in [9] and [10], where a certain minimization problem is solved numerically to get an initial estimate of the MS position, and then a smoothing procedure such as linear regression [9], or the Kalman filter [10] are applied to obtain a more accurate estimate.

In this paper, a MS location and velocity tracking method based on particle filtering is proposed. It employs the instantaneous electric field measurements based on the 3D multipath channel model of Aulin [11] to account for multipath and non-line-of-sight (NLOS) characteristics of the wireless channel as well as the dynamicity of the MS. The received instantaneous electric field in this model is a nonlinear function of the position and velocity of the MS. The generic particle filter (PF) and the unscented particle filter (UPF) are employed to estimate the MS location and velocity. The PF/UPF approaches approximate the optimal solution numerically based on the physical model, rather than applying an optimal filter to an approximate model such as in the extended Kalman filter (EKF). They provide general solutions to many problems where linearization and Gaussian approximations are intractable or yield low performance. The more nonlinear the model is or the more non-Gaussian noise is, the more potential PF/UPF have, especially in applications where computational power is rather cheap and the sampling rate is moderate.

Particle filtering has been used in several tracking wireless applications [12]-[15], but the channel models used do not take into account the NLOS and multipath properties of the wireless channel. In this paper, the proposed algorithms take into account these propertied and require only one base station (BS) to estimate the MS location instead of at least three BSs as found in the literature [10], [16]. The PF approach together with the multipath channel model of Aulin has been presented in [17] for the localization problem. In this paper we present the more recent UPF to estimate the mobile location and velocity, and compare its performance with the generic PF in [17] and the EKF in [16]. Moreover, numerical results are performed under uncertainties in the model parameters to evaluate the robustness of the algorithm.

The paper is structured as follows. In Section II, we describe Aulin mathematical model used for the location and velocity estimation algorithms. The PF and the UPF approaches for MS location and velocity estimation are

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presented in Sections III and IV, respectively. In Section V, we present numerical results, and evaluate the robustness of the proposed algorithms due to uncertainties in the channel parameters. Section VI provides concluding remarks.

II. System Mathematical Model

The basic 3D wireless scattering channel model described in [11], which assumes that the electric field, denoted by $E(t)$, at any receiving point (x_0, y_0, z_0) is the resultant of P plane waves (see Fig. 1), in which the receiver moves in the X-Y plane having velocity v in a direction making an angle γ with the X-axis, is given by:

$$E(t) = \sum_{n=1}^{P} E_n(t) = \sum_{n=1}^{P} r_n \cos\left(\omega_c t + \omega_n t + \theta_n\right) + e(t) \qquad (1)$$

where

$$\omega_n = {2\pi\upsilon}/{\lambda}\left(\cos(\gamma - \alpha_n)\cos\beta_n\right) \qquad (2)$$

and

$$\theta_n = -{2\pi}/{\lambda}\begin{pmatrix} x_0 \cos\alpha_n \cos\beta_n \\ +y_0 \sin\alpha_n \cos\beta_n + z_0 \sin\beta_n \end{pmatrix} + \phi_n \qquad (3)$$

and α_n, β_n are spatial angles of arrival, ω_n is the Doppler shift, θ_n is the phase shift, r_n is the amplitude, ϕ_n is the phase of the nth component, λ is the wavelength, $e(t)$ is a white Gaussian noise, and P is the total number of paths. It can be seen from (2) and (3) that the Doppler and phase shifts depend on the velocity and location of the receiver, respectively. Therefore, the noisy instantaneous received field in (1) depends parametrically on the location and velocity of the receiver and can be used to estimate the MS location and velocity by using the PF/UPF.

Next, we formulate the location estimation as a filtering problem in state-space form [18]. The general form, once discretized, is given by:

$$\begin{aligned} \mathbf{x}_k &= \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{w}_{k-1}) \\ \mathbf{z}_k &= \mathbf{h}(\mathbf{x}_k, \mathbf{v}_k) \end{aligned} \qquad (4)$$

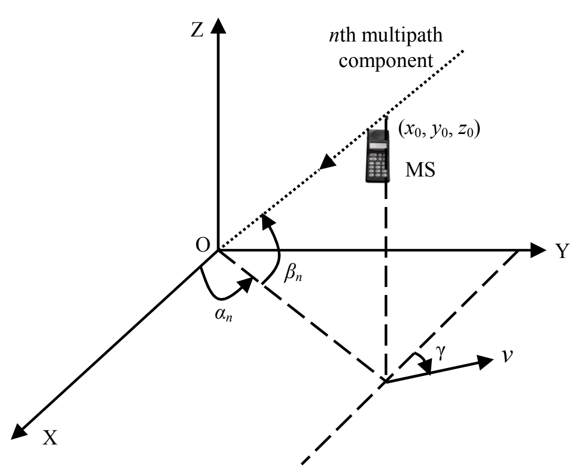


Fig.1. Aulin's 3D multipath channel model.

where $\mathbf{f}(.,.)$ and $\mathbf{h}(.,.)$ are known vector functions, k is the estimation step, $\mathbf{z}_k$ are the output measurements at time step k, and $\mathbf{x}_k$ is the system state at time step k and must not be confused with location coordinates. Further, $\mathbf{w}_k$ and $\mathbf{v}_k$ are the discrete zero-mean, independent state and measurement noise processes, with covariance matrices $\mathbf{Q}$ and $\mathbf{R}$, respectively.

Now let $\mathbf{x}_k = \left[x_k, \dot{x}_k, y_k, \dot{y}_k\right]^T$ denote the state of the MS at time k, where x_k and y_k are the Cartesian coordinates of the MS, $\dot{x}_k$ and $\dot{y}_k$ are the velocities of the MS in the X and Y directions, respectively. We choose the case where the velocity of the MS is not known and is subject to unknown accelerations. The dynamics of the MS are [12]:

$$\begin{bmatrix} x_k \\ \dot{x}_k \\ y_k \\ \dot{y}_k \end{bmatrix} = \begin{bmatrix} 1 & \Delta_k & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta_k \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{k-1} \\ \dot{x}_{k-1} \\ y_{k-1} \\ \dot{y}_{k-1} \end{bmatrix} + \begin{bmatrix} \Delta_k^2/2 & 0 \\ \Delta_k & 0 \\ 0 & \Delta_k^2/2 \\ 0 & \Delta_k \end{bmatrix} \begin{bmatrix} w_{k-1,1} \\ w_{k-1,2} \end{bmatrix} \qquad (5)$$

where Δ_k is a (possibly non-uniform) measurement interval between time $k-1$ and k.

The measurement equation can be found from Aulin's scattering model (1), (2), and (3), which can be written in discrete form as:

$$z_k = h(\mathbf{x}_k, v_k) = \sum_{n=1}^{P} r_{n_k} \cos\left(\omega_c t_k + \omega_{n_k} t_k + \theta_{n_k}\right) + v(t_k) \qquad (6)$$

where

$$\omega_{n_k} = \frac{2\pi\sqrt{\dot{x}_k^2 + \dot{y}_k^2}}{\lambda}\left(\cos(\gamma_k - \alpha_{n_k})\cos\beta_{n_k}\right) \qquad (7)$$

and

$$\theta_{n_k} = \frac{-2\pi}{\lambda}\begin{pmatrix} x_k \cos\alpha_{n_k} \cos\beta_{n_k} \\ +y_k \sin\alpha_{n_k} \cos\beta_{n_k} + z_0 \sin\beta_{n_k} \end{pmatrix} + \phi_{n_k} \qquad (8)$$

Clearly, the measurement equation h(.,.) is a nonlinear function of the state-space vector, as observed in (6), (7), and (8). If we assume knowledge of the channel, which is attainable either through channel estimation at the receiver (e.g., GSM receiver), or through various estimation techniques (e.g., least-squares, ML), then this problem falls under the broad area of nonlinear parameter estimation from noisy data which can be solved using the PF/UPF algorithms. The PF approach is discussed in the next section.

III. The PF Approach for MS Location and Velocity Estimation

Consider the general discrete-time dynamical model described in (4). Let the known probability density functions (PDFs) of the process noise $\mathbf{w}_k$ and the measurement noise $\mathbf{v}_k$ be $p(\mathbf{w}_k)$ and $p(\mathbf{v}_k)$, respectively. As usual, $\mathbf{w}_k$ and $\mathbf{v}_k$ are assumed to be mutually independent. The set of entire measurements from the initial time step to time step k is denoted by $\mathbf{Z}_k = \{\mathbf{z}_i\}_{i=1}^{k}$. The distribution of the initial condition $\mathbf{x}_0$ is assumed to be given by $p(\mathbf{x}_0 \mid \mathbf{Z}_0) = p(\mathbf{x}_0)$.

The PF is a technique for implementing a recursive Bayesian filter by Monte Carlo simulations. The key idea is to represent the required posterior density function by a set of random samples $\{\hat{\mathbf{x}}_k(j)\}_{j=1}^{N}$ with associated weights $\{\omega_k(j)\}_{j=1}^{N}$ and to compute estimates based on these samples and weights. In this case the posterior density at time k can be approximated as:

$$p(\mathbf{x}_k \mid \mathbf{Z}_k) \approx \sum_{j=1}^{N} \omega_k(j)\,\delta\left(\mathbf{x}_k - \hat{\mathbf{x}}_k(j)\right) \tag{9}$$

We therefore have a discrete weighted approximation to the true posterior $p(\mathbf{x}_k \mid \mathbf{Z}_k)$. The weights are chosen using the principle of importance sampling as [19]:

$$\omega_k(j) \propto \frac{p\left(\mathbf{z}_k \mid \tilde{\mathbf{x}}_k(j)\right) p\left(\tilde{\mathbf{x}}_k(j) \mid \tilde{\mathbf{x}}_{k-1}(j)\right)}{q\left(\tilde{\mathbf{x}}_k(j) \mid \tilde{\mathbf{x}}_{k-1}(j), \mathbf{z}_k\right)} \tag{10}$$

where $q\left(\tilde{\mathbf{x}}_k(j) \mid \tilde{\mathbf{x}}_{k-1}(j), \mathbf{z}_k\right)$ is the importance proposal distribution function that generates the samples $\{\hat{\mathbf{x}}_k(j)\}_{j=1}^{N}$. The choice of this distribution function is one of the most critical design issues and determines the type of the PF. The optimal proposal distribution function that minimizes the variance of the weights conditioned on $\tilde{\mathbf{x}}_{k-1}(j)$ and $\mathbf{z}_k$ is $q\left(\mathbf{x}_k \mid \tilde{\mathbf{x}}_{k-1}(j), \mathbf{z}_k\right)_{opt} = p\left(\mathbf{x}_k \mid \tilde{\mathbf{x}}_{k-1}(j), \mathbf{z}_k\right)$ [19].

However, analytical evaluation of the optimal proposal function is not possible for many models, and thus has to be approximated using local linearization [19] or the unscented transformation [20]. In this paper, the unscented transformation method is considered and the resulting filter is called the unscented particle filter (UPF) which is described in Section IV.

Nonetheless, the most popular choice of proposal function is the transition prior $q\left(\mathbf{x}_k \mid \tilde{\mathbf{x}}_{k-1}(j), \mathbf{z}_k\right) = p\left(\mathbf{x}_k \mid \tilde{\mathbf{x}}_{k-1}(j)\right)$. This filter is called the generic PF and is discussed herein. Although this choice of proposal function results in higher Monte Carlo variations than the optimal, it is usually simple to implement.

The time-update stage of the generic PF [21] is performed by passing the random samples $\{\hat{\mathbf{x}}_{k-1}(j)\}_{j=1}^{N}$ through the system model (5) to obtain the time-updated samples $\{\tilde{\mathbf{x}}_k(j)\}_{j=1}^{N}$ as:

$$\tilde{\mathbf{x}}_k(j) = \mathbf{f}\left(\hat{\mathbf{x}}_{k-1}(j), \mathbf{w}_{k-1}(j)\right) \tag{11}$$

where $\mathbf{w}_{k-1}(j)$ is a sample drawn from the PDF $p(\mathbf{w}_{k-1})$ of the system noise. The samples $\{\tilde{\mathbf{x}}_k(j)\}_{j=1}^{N}$ are distributed as the time updated PDF $p(\mathbf{x}_k \mid \mathbf{Z}_{k-1})$.

The measurement-update stage can be described by substituting the choice of proposal distribution $q\left(\mathbf{x}_k \mid \tilde{\mathbf{x}}_{k-1}(j), z_k\right) = p\left(\mathbf{x}_k \mid \tilde{\mathbf{x}}_{k-1}(j)\right)$ into (10) and normalizing which yields:

$$\omega_k(j) = \frac{p\left(z_k \mid \tilde{\mathbf{x}}_k(j)\right)}{\sum_{j=1}^{N} p\left(z_k \mid \tilde{\mathbf{x}}_k(j)\right)} \tag{12}$$

We define a discrete density over $\{\tilde{\mathbf{x}}_k(j)\}_{j=1}^{N}$ with probability mass $\omega_k(j)$ associated with each sample $\tilde{\mathbf{x}}_k(j)$. Then we get the measurement-update samples $\{\hat{\mathbf{x}}_k(j)\}_{j=1}^{N}$ through a resampling process, such that $\Pr\{\hat{\mathbf{x}}_k(i) = \tilde{\mathbf{x}}_k(j)\} = \omega_k(j)$ for any i. Several resampling schemes are presented in the literature such as: systematic [22], stratified, and residual resampling [23]. However, the specific choice of resampling scheme does not significantly affect the performance of the PF. Therefore, systematic resampling is used in all of the experiments in Section VII since it is simple to implement. The estimate of the PF at time k is chosen to be the mean of the samples $\{\hat{\mathbf{x}}_k(j)\}_{j=1}^{N}$.

In the next section, an approximate version of the optimal proposal distribution is considered in order to have a more accurate MS location estimate.

IV. The UPF Approach for MS Location and Velocity Estimation

The UPF results from using a scaled unscented transformation (SUT) method to approximate the optimal proposal distribution within a particle filter framework. The SUT provides more accurate approximation than linearization methods [20]. In particular, the SUT calculates the posterior covariance accurately to the 3rd order, whereas linearization methods such as the EKF rely on a first order biased approximation. The SUT method is introduced next.

A. The SUT Method

The SUT method still approximates the proposal distribution by a Gaussian distribution, but it is specified using a minimal set of deterministically chosen sample points. These sample points completely capture the true mean and covariance of the Gaussian distribution, and when propagated through the true nonlinear system, captures the posterior mean and covariance accurately to the 3rd order for any nonlinearity.

Consider the state equation described in (4). For simplicity, let $\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1})$, where $\mathbf{x}_{k-1}$ is an n_x dimensional random vector and assume $\mathbf{x}_{k-1}$ has mean $\bar{\mathbf{x}}_{k-1}$ and covariance $\mathbf{P}_{k-1}$. Then, a set of $2n_x + 1$ weighted samples or sigma points $S_i = \{W_i, \mathcal{X}_i\}$ are deterministically chosen so that they completely capture the true mean and covariance of the prior random vector $\mathbf{x}_{k-1}$. A selection scheme that satisfies this requirement is [20]:

$$\begin{aligned}
\mathcal{X}_{k-1}^{0} &= \bar{\mathbf{x}}_{k-1} \\
\mathcal{X}_{k-1}^{i} &= \bar{\mathbf{x}}_{k-1} + \left(\sqrt{(n_x+\lambda)\mathbf{P}_{k-1}}\right)_i, \quad i=1,\ldots,n_x \\
\mathcal{X}_{k-1}^{i} &= \bar{\mathbf{x}}_{k-1} - \left(\sqrt{(n_x+\lambda)\mathbf{P}_{k-1}}\right)_i, \quad i=n_x+1,\ldots,2n_x \\
W_0^{(m)} &= \lambda/(n_x+\lambda) \\
W_0^{(c)} &= \lambda/(n_x+\lambda)+\left(1-\alpha^2+\beta\right) \\
W_i^{(m)} &= W_i^{(c)} = 1/\left\{2(n_x+\lambda)\right\}, \quad i=1,\ldots,2n_x
\end{aligned} \tag{13}$$

where $\lambda=\alpha^2(n_x+\kappa)-n_x$, α, β, and κ are scaling parameters, $\left(\sqrt{(n_x+\lambda)\mathbf{P}_{k-1}}\right)_i$ is the ith row or column of the matrix square root of $(n_x+\lambda)\mathbf{P}_{k-1}$, Each sigma point is now propagated through the nonlinear function $\mathcal{X}_k^i=\mathbf{f}(\mathcal{X}_{k-1}^i)$, $i=0,\ldots,2n_x$. And the estimated mean and covariance of $\mathbf{x}_k$ are computed as follows:

$$\begin{aligned}
\bar{\mathbf{x}}_k &= \sum_{i=0}^{2n_x} W_i^{(m)} \mathcal{X}_k^i \\
\mathbf{P}_k &= \sum_{i=0}^{2n_x} W_i^{(c)} \left\{\mathcal{X}_k^i-\bar{\mathbf{x}}_k\right\}\left\{\mathcal{X}_k^i-\bar{\mathbf{x}}_k\right\}^T
\end{aligned} \tag{14}$$

These estimates of the mean and covariance are accurate to the 3rd order for any nonlinear function. In comparison, the EKF only calculates the posterior mean and covariance accurately to the first order with all higher order moments truncated.

B. The UPF Design

The UPF uses the same framework as the regular PF, except that it approximates the optimal proposal distribution by a Gaussian distribution using the SUT method. In particular, the SUT is used to generate and propagate a Gaussian proposal distribution for each particle to get:

$$q\left(\mathbf{x}_k(j) \mid \tilde{\mathbf{x}}_{k-1}(j),\mathbf{z}_k\right)_{opt} \approx \mathcal{N}\left(\bar{\mathbf{x}}_k(j),\mathbf{P}_k(j)\right) \tag{15}$$

where $j=1,\ldots,N$. That is, at time $k-1$ the SUT is used with the new data, to compute the mean and covariance of the importance distribution for each particle. Next, the jth particle is sampled from this distribution.

In the implementation of the UPF, the augmented state vector is defined as the concatenation of the original state and noise variables as $\mathbf{x}_k^a=\left[\mathbf{x}_k^T \; \mathbf{w}_k^T \; v_k\right]^T$. Then the SUT sigma point selection scheme is applied to this new augmented state vector to calculate the corresponding sigma matrix, $\mathcal{X}_k^a$. The complete UPF is described as follows [20]:

a) Initialization $(k=0)$: Draw the particles $\left\{\mathbf{x}_0(j)\right\}_{j=1}^N$ from the prior $p(\mathbf{x}_0)$ and set:

$$\begin{aligned}
\bar{\mathbf{x}}_0(j) &= E\left[\mathbf{x}_0(j)\right] \\
\mathbf{P}_0(j) &= E\left[\left(\mathbf{x}_0(j)-\bar{\mathbf{x}}_0(j)\right)\left(\mathbf{x}_0(j)-\bar{\mathbf{x}}_0(j)\right)^T\right] \\
\bar{\mathbf{x}}_0^a(j) &= E\left[\mathbf{x}_0^a(j)\right]=\left[\left(\bar{\mathbf{x}}_0(j)\right)^T \; 0 \; 0\right]^T \\
\mathbf{P}_0^a(j) &= E\left[\left(\mathbf{x}_0^a(j)-\bar{\mathbf{x}}_0^a(j)\right)\left(\mathbf{x}_0^a(j)-\bar{\mathbf{x}}_0^a(j)\right)^T\right] \\
&= \begin{bmatrix} \mathbf{P}_0(j) & 0 & 0 \\ 0 & \mathbf{Q} & 0 \\ 0 & 0 & \mathbf{R} \end{bmatrix}
\end{aligned} \tag{16}$$

where $E[.]$ is the expectation operator.

b) Now for $k=1,2,\ldots$, the importance sampling step is performed by the following steps:

- Calculating sigma points:

$$\mathcal{X}_{k-1}^a(j)=\left[\bar{\mathbf{x}}_{k-1}^a(j)\bar{\mathbf{x}}_{k-1}^a(j) \pm \sqrt{(n_a+\lambda)\mathbf{P}_{k-1}^a(j)}\right] \tag{17}$$

- Performing the time update stage as:

$$\begin{aligned}
&\tilde{\mathcal{X}}_k^x(j)=\mathbf{f}\left(\mathcal{X}_{k-1}^x(j),\mathcal{X}_{k-1}^v(j)\right), \bar{\tilde{\mathbf{x}}}_k(j)=\sum_{i=0}^{2n_a} W_i^{(m)} \tilde{\mathcal{X}}_{i,k}^x(j) \\
&\tilde{\mathbf{P}}_k(j)=\sum_{i=0}^{2n_a} W_i^{(c)}\left\{\tilde{\mathcal{X}}_{i,k}^x(j)-\bar{\tilde{\mathbf{x}}}_k(j)\right\}\left\{\tilde{\mathcal{X}}_{i,k}^x(j)-\bar{\tilde{\mathbf{x}}}_k(j)\right\}^T \\
&\mathcal{Z}_k(j)=\mathbf{h}\left(\tilde{\mathcal{X}}_k^x(j),\mathcal{X}_{k-1}^n(j)\right), \bar{\tilde{z}}_k(j)=\sum_{i=0}^{2n_a} W_i^{(m)} \mathcal{Z}_{i,k}(j)
\end{aligned} \tag{18}$$

- Performing the measurement update stage as:

$$\begin{aligned}
&\mathbf{P}_{\mathbf{z}_k\mathbf{z}_k}=\sum_{i=0}^{2n_a} W_i^{(c)}\left\{\mathcal{Z}_{i,k}(j)-\bar{\tilde{\mathbf{z}}}_k(j)\right\}\left\{\mathcal{Z}_{i,k}(j)-\bar{\tilde{z}}_k(j)\right\}^T \\
&\mathbf{P}_{\mathbf{x}_k\mathbf{z}_k}=\sum_{i=0}^{2n_a} W_i^{(c)}\left\{\tilde{\mathcal{X}}_{i,k}^x(j)-\bar{\tilde{\mathbf{x}}}_k(j)\right\}\left\{\mathcal{Z}_{i,k}(j)-\bar{\tilde{z}}_k(j)\right\}^T \\
&\mathbf{K}_k=\mathbf{P}_{\mathbf{x}_k\mathbf{z}_k}\mathbf{P}_{\mathbf{z}_k\mathbf{z}_k}^{-1}, \; \bar{\hat{\mathbf{x}}}_k(j)=\bar{\tilde{\mathbf{x}}}_k(j)+\mathbf{K}_k\left(z_k-\bar{\tilde{z}}_k(j)\right) \\
&\hat{\mathbf{P}}_k(j)=\tilde{\mathbf{P}}_k(j)-\mathbf{K}_k\mathbf{P}_{\mathbf{z}_k\mathbf{z}_k}\mathbf{K}_k^T
\end{aligned} \tag{19}$$

and then sampling $\hat{\mathbf{x}}_k(j)$ from $q\left(\mathbf{x}_k(j) \mid \mathbf{x}_{k-1}(j),z_k\right)=\mathcal{N}\left(\bar{\hat{\mathbf{x}}}_k(j),\hat{\mathbf{P}}_k(j)\right)$.

- Evaluating the importance weights as:

$$\omega_k(j) \propto \frac{p\left(z_k \mid \hat{\mathbf{x}}_k(j)\right)p\left(\hat{\mathbf{x}}_k(j) \mid \mathbf{x}_{k-1}(j)\right)}{q\left(\hat{\mathbf{x}}_k(j) \mid \mathbf{x}_{k-1}(j),z_k\right)} \tag{20}$$

and then normalizing the importance weights for $j=1,\ldots,N$.

c) Finally, a resampling process such as systematic resampling is performed to obtain N random particles $\left(\hat{\mathbf{x}}_k(j),\hat{\mathbf{P}}_k(j)\right)$, and the output is generated in the same manner as for the generic PF.

In the next section, numerical examples are presented to illustrate the accuracy of the proposed algorithms.

V. NUMERICAL RESULTS

In this numerical example, the wireless communication network and the PF parameters are similar to the ones in [17]. The SUT parameters are set to $\alpha = 1$, $\beta = 0$, and $\kappa = 0$. The position (or velocity) root mean square error (RMSE) is used as a performance measure and is defined as:

$$\text{RMSE}(k) = \sqrt{\frac{1}{MC}\sum_{i=1}^{MC}\left(\hat{\mathbf{x}}_k^i - \mathbf{x}_k^{\text{true}}\right)^T \left(\hat{\mathbf{x}}_k^i - \mathbf{x}_k^{\text{true}}\right)} \quad (21)$$

where MC is the number of Monte Carlo simulations performed, and $\hat{\mathbf{x}}_k^i$ is the filter position estimate $(x, y)^T$ (or velocity estimate $(\dot{x}, \dot{y})^T$), at time k in Monte Carlo run i. The overall RMSE is defined as:

$$\text{RMSE} = \sqrt{\frac{1}{L}\sum_{k=1}^{L}\frac{1}{MC}\sum_{i=1}^{MC}\left(\hat{\mathbf{x}}_k^i - \mathbf{x}_k^{\text{true}}\right)^T \left(\hat{\mathbf{x}}_k^i - \mathbf{x}_k^{\text{true}}\right)} \quad (22)$$

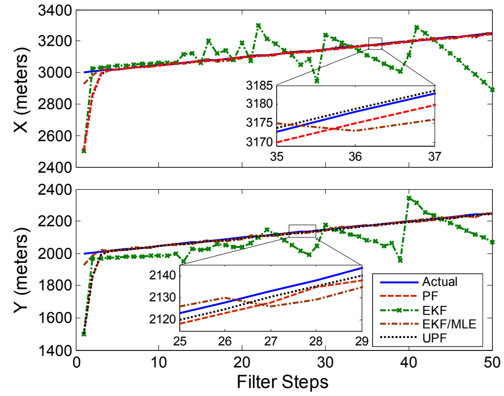
where L is the total number of simulation time steps after the convergence of the filter. The performance of the UPF approach is compared with that of the PF and the EKF approaches in [17] and the maximum likelihood estimation (MLE) approach in [16] which is based on the power path loss channel model.

Fig. 2a and 2b show one realization illustrating the convergence of the proposed and reference algorithms to the real position and velocity of a moving MS, respectively. Fig. 3 shows the position and velocity RMSE for each time according to (21), and the overall position and velocity RMSE for the convergent runs using (22) are shown in Table (1). From Fig. 3 and Table (1), it can be noticed that the accuracy of the MLE approach is satisfactory. However, in realistic NLOS and multipath conditions this method does not perform well since it is based on the power path loss model. Nevertheless, it can be used as initial condition for the EKF to find a more accurate estimator.

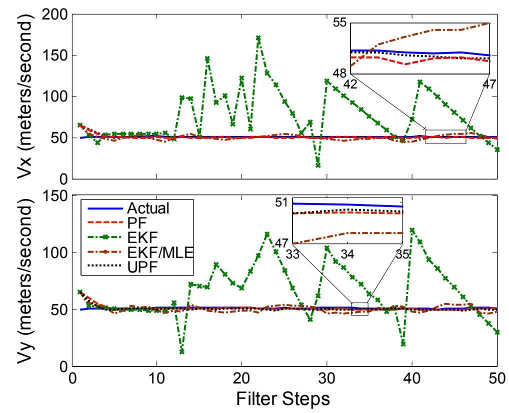
We observe in Fig. 2 that the EKF/MLE, PF, and UPF estimators converge to the actual location and velocity within a few iterations (less than 5). While the EKF position and velocity estimates oscillate with large deviation around the actual position and velocity. This is because the EKF truncates higher order series expansion terms and is sensitive to the initial state. However, the latter can be improved by using the ML estimate as an initial estimate for the EKF. Since it takes less than 5 iterations for the filters to converge near the actual value as shown in Fig. 2, the RMSE (k) in (21) is calculated starting from the iteration k = 5. Only convergent runs are used in the RMSE calculations.

Table 1. Performance comparison: MS location and velocity estimation algorithms using the UPF, the PF, the EKF, the EKF/MLE, and the MLE approaches.

	MLE	EKF	EKF/ MLE	PF	UPF
Diverged runs	–	39	6	2	2
Position RMSE (m)	73.46	142.38	11.23	4.31	3.81
Velocity RMSE (m/sec)	–	51.36	16.52	1.01	0.96



(a)



(b)

Fig. 2. (a) Location and (b) velocity estimates of a moving MS using the UPF, the PF, the EKF, and the EKF/MLE algorithms.

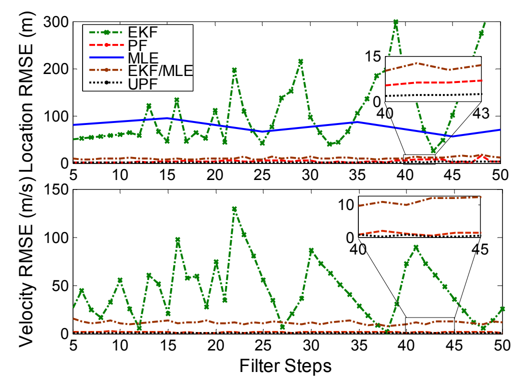


Fig. 3. Location and velocity estimates RMSE (k) using the UPF, the PF, the EKF, and the EKF/MLE algorithms.

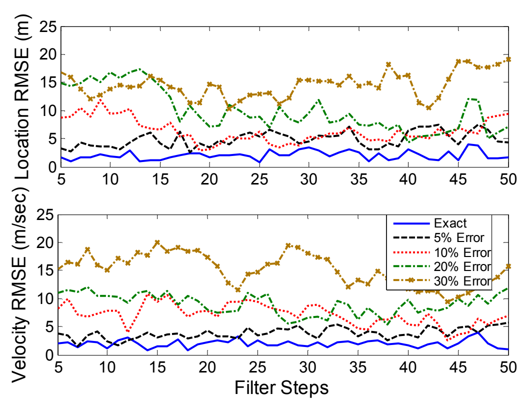


Fig. 4. The UPF location and velocity estimates RMSE (k) for imperfect knowledge of channel parameters.

Fig. 3 shows that the performance of the PF and the UPF approaches are about the same and superior to other approaches. The superior performance of the UPF is clearly evident. Table (1) shows the number of runs that diverged and the performance for each approach. The latter shows the appropriateness of choosing the PF and the UPF for this kind of problems.

Fig. 4 shows how robust the particle filtering approach is if we assume that we only know the channel parameters $\{r_n, \alpha_n, \beta_n\}$ within certain tolerances. Specifically,

$$\begin{aligned} r_n &= r_{n_0}(1+\delta r_{n_0}), & \left|\delta r_{n_0}\right| &\le 5\%, 10\%, 20\% \text{ and } 30\% \\ \alpha_n &= \alpha_{n_0}(1+\delta\alpha_{n_0}), & \left|\delta\alpha_{n_0}\right| &\le 5\%, 10\%, 20\% \text{ and } 30\% \\ \beta_n &= \beta_{n_0}(1+\delta\beta_{n_0}), & \left|\delta\beta_{n_0}\right| &\le 5\%, 10\%, 20\% \text{ and } 30\% \end{aligned} \quad (23)$$

where r_{n_0}, α_{n_0} and β_{n_0} are the nominal (actual) values of the channel parameters. Fig. 4 is generated by assuming that the real channel has parameters r_{n_0}, α_{n_0} and β_{n_0}, while in the estimation stage the channel model parameters used are uniformly distributed about their nominal values as in the uncertainty model (23), and varying the uncertainty percentage from 5% to 30%. It can be noticed that the location and velocity RMSE still converge even if the channel parameters have errors. The higher the error is, the longer time it takes for the filter to converge. It can also be seen that the final RMSE increases for higher errors in channel parameters as expected.

VI. Conclusion

New estimation methods are proposed to track the position and velocity of a MS in a cellular network. They are based on Aulin's scattering model combined with the PF/UPF estimation algorithms. Numerical results for typical simulations including in the presence of parameters uncertainty show that they are highly accurate and consistent. The performance of the PF and the UPF estimation methods are superior to the EKF. The use of nonlinear models and/or non-Gaussian noise is the main explanation for the improvement in accuracy over the EKF. These methods also excel in using inherent features of the cellular system, i.e., they support existing network infrastructure and channel signaling. The assumptions are knowledge of the channel and access to the instantaneous received field, which are obtained through channel sounding samples from the receiver circuitry. Future work will focus on generating efficient channel estimation algorithms, to remove the assumption on partial knowledge of the channel. Work on building a pilot application to test the performance of the PF and/or the UPF in realistic conditions is on-going together with the incorporation of channel model parameters estimation algorithms.

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