Proceedings of the Ninth IASTED International Conference
CONTROL AND APPLICATIONS
May 30–June 1, 2007, Montreal, Quebec, Canada
ISBN Hardcopy: 978-0-88986-665-2 / CD: 978-0-88986-666-9

A GENERAL FRAMEWORK FOR CONTINUOUS TIME POWER CONTROL IN TIME VARYING LONG TERM FADING WIRELESS NETWORKS

Mohammed M. Olama, Seddik M. Djouadi
Electrical and Computer Engineering Department
University of Tennessee
1508 Middle Drive
Knoxville, TN 37996 USA
molama@utk.edu, djouadi@ece.utk.edu

Charalambos D. Charalambous
Electrical and Computer Engineering Department
University of Cyprus
75, Kallipoleos Street, P.O.Box 20537
1678 Nicosia, Cyprus
chadcha@ucy.ac.cy

ABSTRACT
In this paper, a general framework for continuous time power control algorithm under time varying long term fading wireless channels is developed. This contrasts most of the power control algorithms introduced in the literature that assume power control takes place in discrete time intervals and can only be used as long as the time duration for successive adjustments of transmitter powers is less than the coherence time of the channel. In continuous time power control, there is no restriction on how fast the wireless channel is varying. Moreover, a sufficient condition for the existence of the optimal continuous time power is derived. The optimal continuous time power control algorithm is developed under time varying long term fading wireless channels, which are based on stochastic differential equations.

KEY WORDS
Modelling, power control, stochastic differential equation, Banach space, and long term fading

1. Introduction

Power control (PC) is important to improve performance of wireless communication systems. The benefits of power minimization are not just increased battery life, but also increased overall network capacity. Users only need to expand sufficient power for acceptable reception as determined by their quality of service (QoS) specifications that is usually characterized by the signal to interference ratio (SIR) [1]. The majority of research papers in this field use discrete time PC algorithms, which can only be used as long as the time duration for successive adjustments of transmitter powers is less than the coherence time of the channel. In this paper, a general framework for continuous time power control algorithm (PCA) under time varying (TV) long term fading (LTF) wireless channels is developed. In continuous time power control, there is no restriction on how fast the wireless channel is time varying.

PCAs can be classified as centralized and distributed. The centralized PCAs require global out-of-cell information available at base stations. The distributed PCAs require base stations to know only the in-cell information, which can be easily obtained by local measurements. The power allocation problem has been studied extensively as an eigenvalue problem for non-negative matrices [1]-[2], resulting in iterative PCAs that converge each user's power to the minimum power [3]-[5], and as optimization-based approaches [6]. Stochastic PCAs that use noisy interference estimates have been introduced in [7], where conventional matched filter receivers are used. It is shown in [7] that the iterative stochastic PCA, which uses stochastic approximations, converges to the optimal power vector under certain assumptions on the step-size sequence. These results were later extended to the cases when a nonlinear receiver or a decision feedback receiver is used [8]. Much of this previous work deals with discrete time PCAs and static time-invariant channel models. In this paper, a general framework for continuous time PCA under TV wireless channels is developed.

In time-invariant models, channel parameters are random but do not depend on time, and remain constant throughout the observation and estimation phase. This contrasts with TV models, where the channel dynamics become TV stochastic processes [9]-[11]. These models take into account relative motion between transmitters and receivers and temporal variations of the propagating environment such as moving scatterers. They exhibit more realistic behavior of wireless networks. In this paper, we consider dynamical TV LTF channel modelling. The dynamics of LTF wireless channels are captured by stochastic differential equations (SDEs). The SDE model proposed allows viewing the wireless channel as a dynamical system, which shows how the channel evolves in time and space. In addition, it allows well-developed tools of adaptive and non-adaptive estimation and identification techniques (to estimate the model parameters) to be applied to this class of problems [12].

The correct usage of any PCA and thereby the power optimization of the channel models, require the use of TV channel models that capture both temporal and spatial variations of the wireless channel. Since few temporal or even spatio-temporal dynamical models have so far been investigated with the application of any PCA, the suggested dynamical model and PCAs will thus provide a far more realistic and efficient optimal control for wireless channels.

2. Time Varying Lognormal Fading Channel Model

Wireless radio channels experience both long-term fading (LTF) and short-term fading (STF). LTF is modelled by lognormal distributions and STF are modelled by Rayleigh or Ricean distributions [13]. In general, LTF and STF are considered as superimposed and may be treated separately [13]. In this paper, we consider dynamical modelling and power control for LTF channels which are predominate in suburban areas. The STF case has been considered in [10].

The time-invariant power loss (PL) in dB for a given path is given by [13]:

$$PL(d)[dB] = \overline{PL}(d_0)[dB] + 10\alpha \log\left(\frac{d}{d_0}\right) + \tilde{Z} \qquad (1)$$

where $d \geq d_0$, $\overline{PL}(d_0)$ is the average PL in dB at a reference distance d_0 from the transmitter, α is the path loss exponent which depends on the propagation medium, and $\tilde{Z}$ is a zero-mean Gaussian distributed random variable, which represents the variability of the PL due to numerous reflections occurring along the path and possibly any other uncertainty of the propagation environment from one observation instant to the next. In TV LTF models, the PL becomes a random process denoted $\{X(t,\tau)\}_{t\geq 0, \tau\geq\tau_0}$, which is a function of both time t and location represented by τ, where $\tau = d/c$, d is the path length, c is the speed of light, $\tau_0 = d_0/c$ and d_0 is the reference distance. The process $\{X(t,\tau)\}_{t\geq 0, \tau\geq\tau_0}$ represents how much power the signal looses at a particular distance as a function of time. The signal attenuation is defined by $S(t,\tau) \triangleq e^{kX(t,\tau)}$, where $k = -\ln(10)/20$ [13].

The process $X(t,\tau)$ is generated by a mean-reverting version of a general linear SDE given by [9]:

$$\begin{aligned} dX(t,\tau) &= \beta(t,\tau)\big((\gamma(t,\tau) - X(t,\tau)\big)dt \\ &\quad + \delta(t,\tau)dW(t), \\ X(t_0,\tau) &= N\left(\overline{PL}(d)[dB]; \sigma_{t_0}^2\right) \end{aligned} \qquad (2)$$

where $\{W(t)\}_{t\geq 0}$ is a standard Brownian motion (zero drift, unit variance) which is assumed to be independent of $X(t_0,\tau)$, $N(\mu;\kappa)$ denotes a Gaussian random variable with mean μ and variance κ, and $\overline{PL}(d)[dB]$ is the average path loss in dB. The parameter $\gamma(t,\tau)$ models the average TV PL at distance d from transmitter, which corresponds to $\overline{PL}(d)[dB]$ at d indexed by t. This model tracks and converges to this value as time progresses. The instantaneous drift $\beta(t,\tau)\big(\gamma(t,\tau) - X(t,\tau)\big)$ represents the effect of pulling the process towards $\gamma(t,\tau)$, while $\beta(t,\tau)$ represents the speed of adjustment towards this value. Finally, $\delta(t,\tau)$ controls the instantaneous variance or volatility of the process for the instantaneous drift. Define $\{\theta(t,\tau)\}_{t\geq 0} \triangleq \{\beta(t,\tau), \gamma(t,\tau), \delta(t,\tau)\}_{t\geq 0}$. If the random processes in $\{\theta(t,\tau)\}_{t\geq 0}$ are measurable and bounded, then (2) has a unique solution for every $X(t_0,\tau)$ given by [9]:

$$\begin{aligned} X(t,\tau) &= e^{-\beta([t,t_0],\tau)}.\Big\{X(t_0,\tau) \\ &\quad + \int_{t_0}^{t} e^{\beta([u,t_0],\tau)}\big(\beta(u,\tau)\gamma(u,\tau)du + \delta(u,\tau)dW(u)\big)\Big\} \end{aligned} \qquad (3)$$

where $\beta([t,t_0],\tau) \triangleq \int_{t_0}^{t} \beta(u,\tau)du$. This model captures the temporal and spatial variations of the propagation environment as the random parameters $\{\theta(t,\tau)\}_{t\geq 0}$ can be used to model the time and space varying characteristics of the channel.

At every instant of time $X(t,\tau)$ is Gaussian with mean and variance given by:

$$\begin{aligned} E\left[X(t,\tau)\right] &= e^{-\beta([t,t_0],\tau)}. \\ &\quad \left(X_0 + \int_{t_0}^{t} e^{\beta([u,t_0],\tau)}\beta(u,\tau)\gamma(u,\tau)du\right) \\ Var\left[X(t,\tau)\right] &= e^{-2\beta([t,t_0],\tau)}. \\ &\quad \left(\int_{t_0}^{t} e^{2\beta([u,t_0],\tau)}\delta^2(u,\tau)du + \sigma_{t_0}^2\right) \end{aligned} \qquad (4)$$

Moreover, the distribution of $S(t,\tau) = e^{kX(t,\tau)}$ is lognormal with mean and variance given by:

$$
E\left[S(t,\tau)\right]=\exp\left(\frac{2kE\left[X(t,\tau)\right]+k^2Var\left[X(t,\tau)\right]}{2}\right)
$$
$$
Var\left[S(t,\tau)\right]=\exp\left(2kE\left[X(t,\tau)\right]+2k^2Var\left[X(t,\tau)\right]\right) -\exp\left(2kE\left[X(t,\tau)\right]+k^2Var\left[X(t,\tau)\right]\right) \quad (5)
$$

The mean and variance in (4) and (5) show that the statistics of the communication channel vary as a function of both time t and space τ.

In this paper, we consider the uplink channel of a cellular network and we assume that users are already assigned to their base stations. Let M be the number of mobiles (users), and N be the number of base stations. The received signal of the ith mobile at its assigned base station at time t is given by:

$$
y_i(t)=\sum_{j=1}^{M}\sqrt{p_j(t)}s_j(t)S_{ij}(t)+n_i(t) \quad (6)
$$

where $p_j(t)$ is the transmitted power of mobile j at time t, which acts as a scaling on the information signal $s_j(t)$, $n_i(t)$ is the channel disturbance or noise at the base station of mobile i, and $S_{ij}(t)$ is the signal attenuation coefficient between mobile j and the base station assigned to mobile i. Therefore, in a cellular network the spatio-temporal model described in (2) for M mobiles and N base stations can be described as:

$$
\begin{aligned}
dX_{ij}(t,\tau)&=\beta_{ij}(t,\tau)\left(\gamma_{ij}(t,\tau)-X_{ij}(t,\tau)\right)dt \\
&\quad+\delta_{ij}(t,\tau)dW_{ij}(t), \\
X_{ij}(t_0,\tau)&=N\left(\overline{PL}(d)[dB]_{ij};\sigma^2_{t_0}\right),\ 1\le i,\ j\le M
\end{aligned} \quad (7)
$$

and the signal attenuation coefficients $S_{ij}(t,\tau)$ are generated using the relation $S_{ij}(t,\tau)=e^{kX_{ij}(t,\tau)}$, where $k=-\ln(10)/20$. Moreover, correlation between the channels in a multi-user/multi-antenna model can be induced by letting the different Brownian motions W_{ij}'s to be correlated, i.e., $E\left[\mathbf{W}(t)\mathbf{W}(t)^T\right]=\mathbf{Q}(\tau)\cdot t$, where $\mathbf{W}(t)\triangleq\left(W_{ij}(t)\right)$, and $\mathbf{Q}(\tau)$ is some (not necessarily diagonal) matrix that is a function of τ and dies out as τ becomes large.

The TV LTF channel models in (7) are used to generate the link gains of wireless networks for the PCA proposed in the next section.

3. Stochastic Power Control Algorithm

The aim of the PCAs described here is to minimize the total transmitted power of all users while maintaining acceptable QoS for each user. The measure of QoS can be defined by the SIR for each link to be larger than a target SIR. In this section, a PCA is introduced based on the TV LTF channel models derived in the previous section.

Consider the cellular network described in the previous section, the centralized PC problem for *time-invariant* channels can be stated as follows [10]:

$$
\begin{aligned}
&\min_{(p_1\ge0,\dots,p_M\ge0)}\sum_{i=1}^{M}p_i,\quad \text{subject to} \\
&\frac{p_i g_{ii}}{\sum_{j\ne i}^{M}p_j g_{ij}+\eta_i}\ge\varepsilon_i,\quad 1\le i\le M
\end{aligned} \quad (8)
$$

where p_i is the power of mobile i, $g_{ij}>0$ is the *time-invariant* channel gain between mobile j and the base station assigned to mobile i, $\varepsilon_i>0$ is the target SIR of mobile i, and $\eta_i>0$ is the noise power level at the base station of mobile i. The generalization to (8) for the TV LTF channel models in (7), described using path-wise QoS of each user over a time interval $[0,T]$, is given by [14]:

$$
\begin{aligned}
&\min_{(p_1(t)\ge0,\dots,p_M(t)\ge0)}\left\{\sum_{i=1}^{M}p_i(t)\right\},\quad \text{subject to} \\
&\frac{E\left\{p_i(t)s_i^2(t)S_{ii}^2(t)\right\}}{E\left\{\sum_{k\ne i}^{M}p_k(t)s_k^2(t)S_{ik}^2(t)+n_i^2(t)\right\}}\ge\varepsilon_i(t)
\end{aligned} \quad (9)
$$

where $E\{\cdot\}$ is the expectation operator, $t\in[0,T]$, and $i=1,\cdots,M$. Define,

$$
\mathbf{p}(t)\triangleq\begin{pmatrix}p_1(t)\\p_2(t)\\\vdots\\p_M(t)\end{pmatrix},\ \mathbf{u}(t)\triangleq\begin{pmatrix}\frac{\varepsilon_1(t)\eta_1(t)}{E\left[S_{11}^2(t)\right]}\\\frac{\varepsilon_2(t)\eta_2(t)}{E\left[S_{22}^2(t)\right]}\\\vdots\\\frac{\varepsilon_M(t)\eta_M(t)}{E\left[S_{MM}^2(t)\right]}\end{pmatrix},
$$
$$
\mathbf{F}(t)=\begin{cases}0, & i=j\\ \frac{E\left[S_{ij}^2(t)\right]\varepsilon_i(t)}{E\left[S_{ii}^2(t)\right]}, & i\ne j\end{cases} \quad (10)
$$

where $i, j = 1, \cdots, M$. Note that $E\left[S_{ij}^2(t)\right]$ can be calculated from (5) and (4). The constraint in (9) can be rewritten as:

$$\left(\mathbf{I} - \mathbf{F}(t)\right)\mathbf{p}(t) \geq \mathbf{u}(t) \tag{11}$$

If the power control problem in (9) is feasible, then the optimal power satisfies:

$$\mathbf{p}^*(t) = \mathbf{F}(t)\mathbf{p}^*(t) + \mathbf{u}(t) \tag{12}$$

where $\mathbf{p}^*(t)$ is the optimal power vector. Expression (12) shows that the optimal power is the fixed point of the following function:

$$\mathbf{\Phi}(\mathbf{p})(t) \triangleq \mathbf{F}(t)\mathbf{p}(t) + \mathbf{u}(t) \tag{13}$$

The power vector $\mathbf{p}(t)$ is assumed to be continuous and bounded as functions from $[0, T]$ to $\mathbf{R}^{+M}$, that is, it belongs to the Banach space of continuous and bounded functions defined on $[0, T]$, which is denoted by $\mathbf{C}_b([0,T];\mathbf{R}^M)$, under the supremum norm given by:

$$\left\|\mathbf{p}(t)\right\|_\infty = \sup_{0 \leq t \leq T} \left(\sum_{i=1}^{M} \left|p_i(t)\right|^2\right)^{1/2} \tag{14}$$

Assuming $\mathbf{u}(t) \in \mathbf{C}([0,T];\mathbf{R}^M)$, then the map $\mathbf{\Phi}(\cdot)$ is defined as:

$$\mathbf{\Phi}(\cdot):\ \mathbf{C}\left([0,T];\mathbf{R}^M\right) \ \to\ \mathbf{C}\left([0,T];\mathbf{R}^M\right) \tag{15}$$

The existence of a fixed point for $\mathbf{\Phi}(\cdot)$ is guaranteed by the contraction mapping theorem or Banach's fixed-point theorem [15], which states that if

$$\left\|\mathbf{\Phi}(\mathbf{p}_1)(t) - \mathbf{\Phi}(\mathbf{p}_2)(t)\right\|_\infty \leq k \left\|\mathbf{p}_1(t) - \mathbf{p}_2(t)\right\|_\infty \tag{16}$$

for some $k < 1$ and all $\mathbf{p}_1(t), \mathbf{p}_2(t) \in \mathbf{C}\left([0,T];\mathbf{R}^M\right)$, then $\mathbf{\Phi}(\cdot)$ has a uniform fixed point. Expression (16) can be rewritten as:

$$\begin{gathered}\left\|\mathbf{F}(t)\mathbf{p}_1(t) - \mathbf{F}(t)\mathbf{p}_2(t)\right\|_\infty \leq k \left\|\mathbf{p}_1(t) - \mathbf{p}_2(t)\right\|_\infty, \\ \forall\ \mathbf{p}_1(t), \mathbf{p}_2(t) \in \mathbf{C}\left([0,T];\mathbf{R}^M\right)\end{gathered} \tag{17}$$

which is equivalent to:

$$\begin{gathered}\frac{\left\|\mathbf{F}(t)\left(\mathbf{p}_1(t) - \mathbf{p}_2(t)\right)\right\|_\infty}{\left\|\mathbf{p}_1(t) - \mathbf{p}_2(t)\right\|_\infty} \leq k < 1, \\ \forall\ \mathbf{p}_1(t), \mathbf{p}_2(t) \in \mathbf{C}\left([0,T];\mathbf{R}^M\right)\end{gathered} \tag{18}$$

Expression (8) holds if and only if the following holds:

$$\sup_{\substack{\mathbf{p}_1(t) \neq \mathbf{p}_2(t) \\ \mathbf{p}_1(t), \mathbf{p}_2(t) \in \mathbf{C}\left([0,T];\mathbf{R}^M\right)}} \frac{\left\|\mathbf{F}(t)\left(\mathbf{p}_1(t) - \mathbf{p}_2(t)\right)\right\|_\infty}{\left\|\mathbf{p}_1(t) - \mathbf{p}_2(t)\right\|_\infty} \leq k < 1 \tag{19}$$

The LHS in (19) is equal to the induced norm of $\mathbf{F}(t)$ viewed as a multiplication acting from $\mathbf{C}([0,T];\mathbf{R}^M)$ into $\mathbf{C}([0,T];\mathbf{R}^M)$, i.e.,

$$\left\|\mathbf{F}(t)\right\| \leq k < 1 \tag{20}$$

where

$$\left\|\mathbf{F}(t)\right\| = \sup_{\substack{\mathbf{p}(t) \in \mathbf{C}\left([0,T];\mathbf{R}^M\right) \\ \left\|\mathbf{p}(t)\right\|_\infty \leq 1}} \left\|\mathbf{F}(t)\mathbf{p}(t)\right\|_\infty \tag{21}$$

$\left\|\mathbf{F}(t)\right\|$ is equal to supremum with respect to t of the largest singular value of $\mathbf{F}(t)$, that is

$$\left\|\mathbf{F}(t)\right\| = \sup_{t \in [0,T]} \bar{\sigma}\left(\mathbf{F}(t)\right) < 1 \tag{22}$$

where $\bar{\sigma}(\cdot)$ denotes the largest singular value of $\mathbf{F}(t)$. Expression (22) gives a sufficient condition on the channels' attenuation coefficients for the existence of an optimal power. Expression (22) is satisfied if and only if:

$$\bar{\sigma}\left(\mathbf{F}(t)\right) < 1, \quad \forall\ t \in [0, T] \tag{23}$$

Thus, if (23) is satisfied, the following continuous time PCA will converge to the minimal power:

$$\mathbf{p}_{k+1}(t) = \mathbf{F}(t)\mathbf{p}_k(t) + \mathbf{u}(t) \tag{24}$$

Note that in (24) index k is iteration on the continuous time power vector and therefore does not represent the time variable as in most PCAs in the literature. Numerical results are presented in the next section.

4. Numerical Results

The LTF cellular network has the following features:

- The number of transmitters (mobiles) is $M = 24$.

- The information signal $s_i(t) = 1$ for $i = 1,...,M$.
- Target SIR, $\varepsilon_i(t) = 5$ for $i = 1,...,M$.
- The transmitted powers are computed every 1 second, and the simulation is performed for 2 minutes.
- Initial distances of all mobiles with respect to their own base stations are generated as uniformly independent identically distributed (i.i.d.) random variables (r.v.'s) in [10 – 100] meters.
- Cross initial distances of all mobiles with respect to other base stations are generated as uniformly i.i.d. r.v.'s in [250 - 550] meters.
- The angle between the direction of motion of mobile j and the distance vector passes through base station *i* and the mobile *j* are generated as uniformly i.i.d. r.v.'s in [0 – 180] degrees.
- The average velocities of mobiles are generated as uniformly i.i.d. r.v.'s in [40 – 100] km/hr.
- PL exponent is 3.5.
- Initial reference distance from each of the transmitters is 10 m.
- PL at the initial reference distance is 15 dB.
- $\delta(t)$ = 1400 and $\beta(t)$ = 225000 for the SDE's.
- η_i's for $i = 1,...,M$ are i.i.d. Gaussian r.v.'s with zero mean and variance = 10^{-12} W.

Figure 1 shows the total transmitted power of all mobiles using the proposed PCA in (24) for fixed time instant (t = 40 seconds) under the stochastic TV LTF wireless network described above. It can be noticed that the total power converges to the optimal power after few iterations even if the initial value is not close to the optimal one.

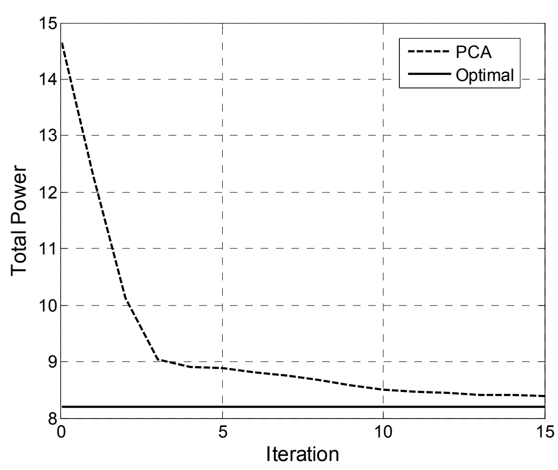

Fig. 1. Sum of transmitted power of all mobiles for the proposed PCA under TV LTF wireless network.

Figure 2 shows the total transmitted power of all mobiles using the proposed PCA in (24) as a function of time. It can be seen that the total power varies rapidly as a function of time, since the mobiles move in different directions and velocities.

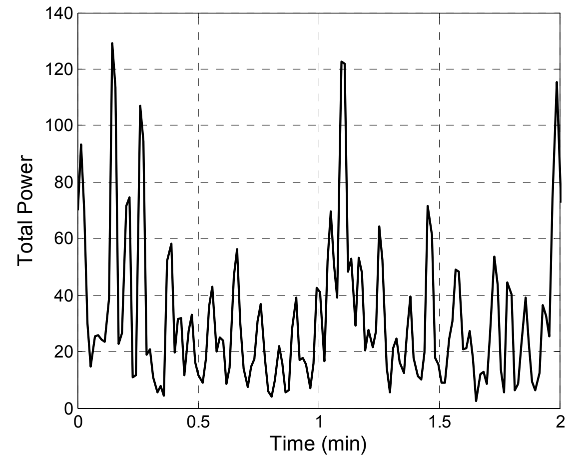

Fig. 2. Sum of transmitted power of all mobiles with respect to time for the proposed PCA under TV LTF wireless network.

5. Conclusion

A general framework for continuous time power control algorithm under time varying long term fading wireless channels is developed. The channel models are represented by 1st order stochastic differential equations (SDEs). The SDE models proposed allow viewing the wireless channel as a dynamical system, which shows how the channel evolves in time and space. In addition, it allows well-developed tools of adaptive and non-adaptive estimation and identification techniques (to estimate the model parameters) to be applied to this class of problems. A sufficient condition on the time varying channels' attenuation coefficients for the existence of the optimal continuous time power is derived. Since the proposed power control algorithm considers continuous time power control, there is no restriction on how fast the wireless channel is varying. Numerical results show that the proposed iterative power control algorithm converges to the optimal power. Future work will focus on deriving sufficient condition on the channel parameters $\{\theta(t,\tau)\}_{t\geq 0}$ for the existence of the optimal continuous time power vector.

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