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Stochastic Channel Modeling for Ad Hoc Wireless Networks

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Abstract— **Due to nodes mobility and environmental changes in mobile ad hoc networks, the ad hoc channel is time varying and subject to fading. As a consequence of these variations, the statistical characteristics of the received signal vary continuously, giving rise to a Doppler power spectral density (DPSD) which varies from one observation instant to the next. Therefore, the traditional models can no longer capture and track complex time variations in the propagation environment. These time variations compel us to introduce more advanced dynamical models in order to capture higher order dynamics of the ad hoc channel. Stochastic ad hoc channel models, in which the evolution of the dynamical channels is described by a stochastic state space representation, are derived. The parameters of the stochastic state space models are determined by approximating the band limited DPSD. Two approximating methods are considered. The first one is simple and the second one is the complex cepstrum method. Inphase and quadrature components of the proposed stochastic ad hoc channel models are derived. Numerical results show that link performance for ad hoc case is worse than cellular case, but the performance gap shrinks with increased mobility.**

I. INTRODUCTION

MOBILE ad hoc and sensor networks comprise nodes that freely and dynamically self-organize into arbitrary and temporary network topology without any infrastructure support [1]. They are self-configured wireless networks, where the information travels from a source node to a destination node by using other nodes in the network as relays [2]. Thus, those networks are formed without any pre-designed infrastructure or centralized administration, in contrast to cellular networks. Copious ad hoc networking research exists on layers in the open system interconnection (OSI) model above the physical layer. However, neglecting the physical layer while modeling wireless environment is error prone and should be considered more carefully [3].

The experimental results in [4] show that the factors at the physical layer not only affect the absolute performance of a protocol, but because their impact on different protocols is non-uniform, it can even change the relative ranking among protocols for the same scenario. It is demonstrated in [5] the importance of the physical layer when evaluating Medium Access Control (MAC) performance.

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Most of the research that takes into accounts the physical layer considerations, such as [6, 7], deals mainly with traditional wireless ad hoc channel models. In these models the speed of sensors are assumed to be constant, and the statistical characteristics of the received signal are assumed to be fixed with respect to time, in which the Doppler power spectral density (DPSD) is fixed from one observation instant to the next. But in reality, the propagation environment varies continuously due to mobility of the nodes at variable speeds causing network topology to dynamically change, the angle of arrival of the wave upon the receiver can vary continuously, and objects or scatters move in between the transmitter and the receiver resulting in appearance or disappearance of existing paths from one instant to the next. As a consequence of these variations, the statistical characteristics of the received signal vary continuously, giving rise to a DPSD which varies from one observation instant to the next. Therefore, the traditional models that assume fixed statistics can no longer capture and track complex time variations in the propagation environment. These time variations compel us to introduce more advanced dynamical models in order to capture higher order dynamics of the ad hoc channel.

In this paper, the DPSD derived in [7] will be used to develop a dynamical stochastic state space models for short term fading (STF) ad hoc channel, which involves the inphase and quadrature components as stochastic processes. The random variables characterizing the instantaneous power in static channel models are generalized to dynamical models including random processes with time varying (TV) statistics. Inphase and quadrature components of the TV ad hoc channel are derived from the stochastic state space models. The parameters of the stochastic state space models can be estimated on-line from the DPSD measurements. Since these models are based on state space representations, they allow the tools of system theory, identification, and estimation techniques to be applied to this class of problems. Time domain simulations using the developed stochastic state space models produce the inphase, quadrature, and therefore the attenuation coefficient of the ad hoc channel. Numerical results are provided to evaluate the performance of the proposed ad hoc channel models.

The paper is organized as follows. In section II, the DPSD of ad hoc channels is introduced as described in [7]. In section III, the TV channel impulse response is discussed. Approximating the band-limited ad hoc DPSD is discussed in section IV. In section V, the stochastic ad hoc channel models are derived. Numerical results and link performance are shown in section VI. Finally section VII gives the conclusion of the work developed in this paper.

II. DPSD OF AD HOC CHANNELS

Dependent on mobile speed, wavelength, and angle of incidence, the Doppler frequency shifts on the multipath rays give rise to a DPSD. The one-sided "cellular" DPSD for a received fading carrier of frequency f_c is given by [8]:

$$\frac{S(f)}{pG/\pi f_{m_{1\to}}} = \begin{cases} \dfrac{1}{\sqrt{1-\left(\dfrac{f-f_c}{f_{m_{1\to}}}\right)^2}}, & |f-f_c| < f_{m_{1\to}} \\ 0 & \text{, otherwise} \end{cases} \quad (1)$$

where $f_{m_{1\to}} = (v/c) f_c$ is the maximum Doppler shift (ignoring second and higher order effects), p is the average power received by an isotropic antenna, G is the gain of the receiving antenna, v is speed of mobile, and c is speed of light. The DPSD represents the frequency response of the channel as well as time variations experienced by the channel. From now on, regarding the one-sided "ad hoc" DPSD, let $f_c = 0$, $f_{m_{1\to}} = 1$, and $pG = \pi$ in (1), solely for mathematical convenience. For a link, with v_1 and v_2 as respectively the sender and receiver's speeds, the *degree of double mobility*, denoted by α, is defined to be $\alpha = \left[\min(v_1, v_2)/\max(v_1, v_2)\right]$, so $0 \le \alpha \le 1$, where $\alpha = 1$ for full double mobility and $\alpha = 0$ for single mobility like the cellular link [7].

Now consider a multihop ad hoc link over n nodes, the classical ($v_i \ll c$) maximum Doppler shift is given by [7]:

$$f_{m_{1\to 2,2\to\ldots\to n}} = \left(v_1 + 2\sum_{i=2}^{n-1} v_i + v_n\right)\frac{f_c}{c} \quad (2)$$

and if there is a mechanism to remove Doppler spread at each relay node, which is usually the case in practice where coherent demodulation takes place after each single hop, the maximum Doppler shift becomes:

$$f_{m_{1\to 2,2\to\ldots\to n}} = \left(v_{n-1} + v_n\right)\frac{f_c}{c} \quad (3)$$

Since, as stated above, it is more practical to consider Doppler effects for a single hop, the ad hoc DPSD is derived for two nodes with the same speed $(\alpha = 1)$, without loss of generality as [6, 7]:

$$S_2(f) = \int_{-1}^{1} S(f-x) S(x)\, dx = \mathrm{K}\left(\sqrt{1-\left(\frac{f}{2}\right)^2}\right) \quad (4)$$

where $\mathrm{K}(.)$ is the complete elliptic integral of the first kind [8]. For $\alpha = 1$ and arbitrary f_c, p, G, and $f_{m_{1\to}} = (1/2) f_{m_{1\to 2}}$, the ad hoc DPSD, $S_2(f)$, is given by:

$$\frac{S_2(f)}{(pG)^2/\pi^2 f_{m_{1\to}}^2} = \begin{cases} \mathrm{K}\left(\sqrt{1-\left(\dfrac{f-f_c}{2 f_{m_{1\to}}}\right)^2}\right), & |f-f_c| < 2 f_{m_{1\to}} \\ 0 & \text{, otherwise} \end{cases} \quad (5)$$

The ad hoc DPSD is used in the next section to derive a method based on stochastic differential equations (SDEs) for the inphase and quadrature components of the ad hoc channel via approximations by rational functions.

III. REPRESENTATION OF TIME VARYING CHANNELS

The general TV model of wireless channels is typically represented by the following multipath band-pass impulse response [9]:

$$C(t;\tau) = \sum_{n=1}^{N(t)} \left(I_n(t,\tau)\cos\omega_c t - Q_n(t,\tau)\sin\omega_c t\right)\delta\left(\tau - \tau_n(t)\right) \quad (6)$$

where $C(t;\tau)$ is the band-pass response of the channel at time t, due to an impulse applied at time $t-\tau$, $N(t)$ is the random number of multipath components, ω_c is the carrier frequency, and the set $\left\{I_n(t,\tau), Q_n(t,\tau), \tau_n(t)\right\}_{n=1}^{N(t)}$ describes the random TV inphase component, quadrature component, and arrival time of the different paths, respectively. Let $s_l(t)$ be the low pass equivalent representation of the transmitted signal, then the band pass representation of the received signal is given by:

$$y(t) = \sum_{n=1}^{N(t)} \left(I_n(t,\tau)\cos\omega_c t - Q_n(t,\tau)\sin\omega_c t\right) s_l\left(t - \tau_n(t)\right) \quad (7)$$

The main idea in constructing the dynamical models for STF ad hoc channels is to factorize the DPSD into an approximate nth order even transfer function, and then any stochastic realization can be used to obtain a state space representation for inphase and quadrature components.

The ad hoc channel is considered as a dynamical system for which the input-output map is described in (6). Note that $S_2(\omega) = |H(\omega)|^2$, where $H(\omega)$ is the Fourier transform of the impulse response of the channel. In general, in order to be able to describe $S_2(\omega)$ in the form of a stochastic differential equation (SDE), we need to find a transfer function, $H(\omega)$, whose magnitude square equals $S_2(\omega)$. This is an old problem which had been studied by Paley and Wiener [10] and is reformulated here as follows: Given a non-negative integrable function, $S(\omega)$, such that the Paley-Wiener condition $\int_{-\infty}^{\infty}\left[\left|\log S(\omega)\right|/\left(1+\omega^2\right)\right] d\omega < \infty$ is satisfied, then there exists a causal, stable, minimum-phase, $H(s)$, such that $|H(\omega)|^2 = S(\omega)$, implying that $S(\omega)$ is factorizable, namely, $S(s) = H(s)H(-s)$. The factor $H(s)$ represents the frequency response of a causal, stable, and minimum-phase system. It can be seen that the Paley-Wiener condition is not satisfied if $S(\omega)$ is band-limited. Note that the DPSD of an ad hoc link is band-limited. Therefore, the DPSD has to be first approximated by a rational transfer function, $\tilde{S}(\omega)$. The approximation procedure will be discussed in the next section.

IV. Approximating the Ad Hoc DPSD

Any approximation method can be used to approximate the ad hoc DPSD. The choice of which depends on the accuracy required. In this paper, two approximation methods will be presented. The first one is simple so it is easy to implement, and the second one is more accurate but requires more computations.

A. *Simple Approximation Method*

In the first method a 4th order even transfer function $\tilde{S}(\omega)$, is used to approximate the ad hoc DPSD, $S(\omega)$. The approximate function $\tilde{S}(s) = H(s)H(-s)$ is given by:

$$\tilde{S}(s) = \frac{K^2}{s^4 + 2\omega_n^2(1-2\zeta^2)s^2 + \omega_n^4}, H(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (8)$$

where $\tilde{S}(s)$ is the approximation of $S(s)$. Equation (8) has three arbitrary parameters $\{\zeta, \omega_n, K\}$, which can be adjusted such that the approximate curve coincides with the actual curve at different points. The reason for presenting 4th order approximation of the DPSD is that we can compute explicit expressions for the constants $\{\zeta, \omega_n, K\}$ as functions of specific points on the data-graphs of the DPSD. In fact, if these parameters are chosen such that:

$$\zeta = \sqrt{\frac{1}{2}\left(1 - \sqrt{1 - \frac{S(0)}{S(j\omega_{\max})}}\right)}, \omega_n = \frac{\omega_{\max}}{\sqrt{1-2\zeta^2}}, K = \omega_n^2\sqrt{S(0)} \quad (9)$$

then the approximate density $\tilde{S}(s)$ coincides with the exact density $S(s)$ at $\omega=0$ and $\omega=\omega_{\max}$. Figure (1) shows the DPSD, $S(f)$, and its approximation $\tilde{S}(f)$ via 4th order even function for different α's using the simple approximation method.

The order of approximation dictates how close the approximate curve would be to the actual one. Higher order approximations can capture higher order dynamics, and while they provide indisputably better approximations for the DPSD, computations become more involved. A more accurate approximating method is discussed next.

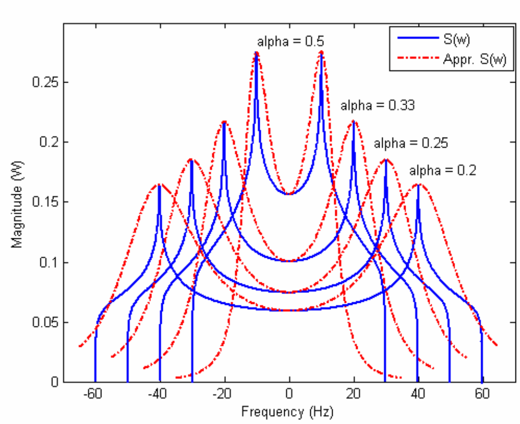

Fig. 1. DPSD, $S(f)$, and its approximation, $\tilde{S}(f)$, using the simple approximation method via 4th order function for different α's.

B. *Complex Cepstrum Approximation Method*

The second approximation method, which is the complex cepstrum algorithm [11], uses all the measured points of the DPSD instead of just three points as in the first method. It is used to generate a stable, minimum phase, real rational transfer function. It can be explained briefly as follows: On a log-log scale, the magnitude data is interpolated linearly, with a very fine discretization. Then, using the complex cepstrum algorithm [11], the phase, associated with a stable, minimum phase, real, rational transfer function with the same magnitude as the magnitude data is generated. With the new phase data and the input magnitude data, a real rational transfer function can be found by using an equation error method to identify the best model from the data as:

$$\min_{b,a} \sum_{k=1}^{l} wt(\omega_k)\left|H(\omega_k)D(\omega_k) - N(\omega_k)\right|^2 \quad (10)$$

where

$$H(s) = \frac{N(s)}{D(s)} = \frac{b_n s^n + b_{n-1}s^{n-1} + \ldots + b_1 s + b_0}{a_m s^m + a_{m-1}s^{m-1} + \ldots + a_1 s + a_0} \quad (11)$$

$b = \{b_n, b_{n-1}, \ldots, b_0\}$, $a = \{a_m, a_{m-1}, \ldots, a_0\}$, $n \le m$, $wt(\omega)$ is the weight function, and l is the number of frequency points. Several variants have been suggested in the literature, where the weighting function gives less attention to high frequencies. This algorithm is based on Levi [12].

For more accurate results, the superior ("output-error") algorithm uses the damped Gauss-Newton method for iterative search [13], with the output of the equation error method as the initial estimate. This solves the direct problem of minimizing the weighted sum of the squared error between the actual and the desired frequency response points as:

$$\min_{b,a} \sum_{k=1}^{l} wt(\omega_k)\left|H(\omega_k) - \frac{N(\omega_k)}{D(\omega_k)}\right|^2 \quad (12)$$

Figure (2a) shows the DPSD, $S(f)$, and its approximation $\tilde{S}(f)$ via different orders using complex cepstrum algorithm. Higher order of $\tilde{S}(f)$, better approximation obtained. It can be seen that approximating by a 4th order transfer function gives very good results. Figure (2b) shows the DPSD, $S(f)$, and its approximation $\tilde{S}(f)$ using the complex cepstrum approximation method for different α's via 4th order even function. It can be noticed that this method gives better approximation than the simple method; since it employs all measured points of the DPSD instead of just three points in the simple method.

It should be mentioned that any satisfactory approximation algorithm could be used to approximate the ad hoc DPSD. The choice depends on the computational cost and the required accuracy. In the next section, the approximated ad hoc DPSD is used to develop stochastic ad hoc channel models.

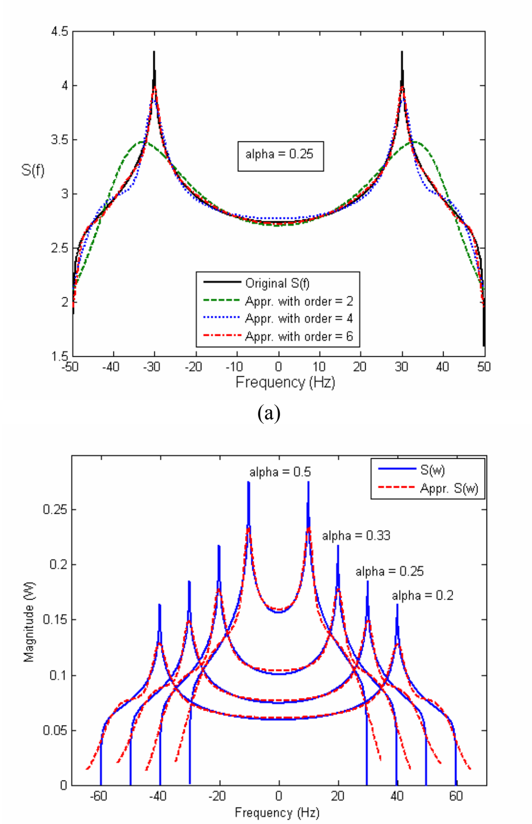

Fig. 2. DPSD, $S(f)$, and its approximations, $\tilde{S}(f)$, using complex cepstrum algorithm for (a) different orders of $\tilde{S}(f)$ (b) 4th order $\tilde{S}(f)$.

V. Stochastic Ad Hoc Channel Models

First, the stochastic ad hoc channel models are derived using the first (simple) approximation method. Then the same procedure will be used to derive stochastic models for the second (complex cepstrum) approximation method.

A. Stochastic Ad Hoc Channel Models Using the Simple Approximation Method

The SDE, which corresponds to $H(s)$ in (8) is given by:

$$\ddot{x}(t)+2\zeta\omega_n\dot{x}(t)+\omega_n^2 x(t)=K\dot{w}(t) \tag{13}$$

where $\dot{x}(0), x(0)$ are given, $\{\dot{w}(t)\}_{t\geq 0}$ is a white-noise process. Equation (13) can be rewritten in terms of inphase and quadrature components as:

$$\begin{aligned}&\ddot{x}_I(t)+2\zeta\omega_n\dot{x}_I(t)+\omega_n^2 x_I(t)=K\dot{w}_I(t),\\&\ddot{x}_Q(t)+2\zeta\omega_n\dot{x}_Q(t)+\omega_n^2 x_Q(t)=K\dot{w}_Q(t),\\&\dot{x}_I(0),x_I(0),\dot{x}_Q(0),x_Q(0)\text{ are given}\end{aligned} \tag{14}$$

where $\{\dot{w}_I(t)\}_{t\geq 0}$ and $\{\dot{w}_Q(t)\}_{t\geq 0}$ are two independent and identically distributed (*i.i.d.*) white Gaussian noises.

Any stochastic realization can be used to obtain a state-space representation for the in-phase and quadrature components of STF ad hoc channel models. For example, the controllable canonical form (CCF) realization can be used to realize (14) for the inphase and quadrature components as:

$$\begin{aligned}&\begin{bmatrix}dX_{I,j}^1(t)\\dX_{I,j}^2(t)\end{bmatrix}=\begin{bmatrix}0 & 1\\-\omega_n^2 & -2\xi_n\omega_n\end{bmatrix}\begin{bmatrix}X_{I,j}^1(t)\\X_{I,j}^2(t)\end{bmatrix}dt\\&\qquad+\begin{bmatrix}0\\K\end{bmatrix}dW_j^I(t),\quad\begin{bmatrix}X_{I,j}^1(0)\\X_{I,j}^2(0)\end{bmatrix},\\&I_j(t)=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}X_{I,j}^1(t)\\X_{I,j}^2(t)\end{bmatrix}+f_j^I(t),\\&\begin{bmatrix}dX_{Q,j}^1(t)\\dX_{Q,j}^2(t)\end{bmatrix}=\begin{bmatrix}0 & 1\\-\omega_n^2 & -2\xi_n\omega_n\end{bmatrix}\begin{bmatrix}X_{Q,j}^1(t)\\X_{Q,j}^2(t)\end{bmatrix}dt\\&\qquad+\begin{bmatrix}0\\K\end{bmatrix}dW_j^Q(t),\quad\begin{bmatrix}X_{Q,j}^1(0)\\X_{Q,j}^2(0)\end{bmatrix},\\&Q_j(t)=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}X_{Q,j}^1(t)\\X_{Q,j}^2(t)\end{bmatrix}+f_j^Q(t)\end{aligned} \tag{15}$$

where $I_j(t)$ and $Q_j(t)$ corresponds to the inphase and quadrature components of the *j*th path respectively, $\{W_j^I(t)\}_{t\geq 0}$ and $\{W_j^Q(t)\}_{t\geq 0}$ are independent standard Brownian motions, which correspond to the inphase and quadrature components respectively, of the *j*th path, the parameters $\{\zeta,\omega_n,K\}$ are obtained from the approximation of the DPSD, $f_j^I(t)$ and $f_j^Q(t)$ are arbitrary functions representing the specular (or LOS) of the inphase and quadrature components respectively, characterizing further dynamic variations in the environment. Note that the parameters $\{\zeta,\omega_n,K\}$ are in reality TV and have to be estimated on-line from the DPSD or time domain measurements.

Let us denote $\{X_I(t),X_Q(t)\}$ the state vectors for the inphase and quadrature components respectively, and $\{I(t),Q(t)\}$ the in-phase and quadrature components of the channel, then (15) for the *j*th path can be written as:

$$\begin{aligned}&\begin{bmatrix}dX_I(t)\\dX_Q(t)\end{bmatrix}=\begin{bmatrix}A_I & 0\\0 & A_Q\end{bmatrix}\begin{bmatrix}X_I(t)\\X_Q(t)\end{bmatrix}dt\\&\qquad+\begin{bmatrix}b_I & 0\\0 & b_Q\end{bmatrix}\begin{bmatrix}dW_I(t)\\dW_Q(t)\end{bmatrix},\quad\begin{bmatrix}X_I(0)\\X_Q(0)\end{bmatrix}\\&\begin{bmatrix}I(t)\\Q(t)\end{bmatrix}=\begin{bmatrix}C_I & 0\\0 & C_Q\end{bmatrix}\begin{bmatrix}X_I(t)\\X_Q(t)\end{bmatrix}\\&\qquad+\begin{bmatrix}D_I & 0\\0 & D_Q\end{bmatrix}\begin{bmatrix}dW_I(t)\\dW_Q(t)\end{bmatrix}+\begin{bmatrix}f_I(t)\\f_Q(t)\end{bmatrix}\end{aligned} \tag{16}$$

where

$$\begin{aligned}&A_I=A_Q=\begin{bmatrix}0 & 1\\-\omega_n^2 & -2\xi_n\omega_n\end{bmatrix},\ b_I=b_Q=\begin{bmatrix}0\\K\end{bmatrix}\\&C_I=C_Q=\begin{bmatrix}1 & 0\end{bmatrix},\ D_I=D_Q=0\end{aligned} \tag{17}$$

$\{W_I(t), W_Q(t)\}_{t\geq 0}$ are independent standard Brownian motions which are independent of the initial random variables $X_I(0)$ and $X_Q(0)$, and $\{f_I(s), f_Q(s); 0 \leq s \leq t\}$ are random processes represent the inphase and quadrature LOS components, respectively.

B. Stochastic Ad Hoc Channel Models Using Complex Cepstrum Approximation Method

Now, we will derive the stochastic ad hoc models by considering the second approximation method. Note that (11) can be written as:

$$H(s) = \frac{b'_{m-1}s^{m-1} + \ldots + b'_1 s + b'_0}{s^m + a'_{m-1}s^{m-1} + \ldots + a'_1 s + a'_0} + D \quad (18)$$

where D is a constant. Following the same procedure as discussed earlier in deriving stochastic models using the first approximation method, the stochastic CCF of (18) for the inphase components is:

$$\begin{bmatrix} dX^1_{I,j}(t) \\ dX^2_{I,j}(t) \\ \vdots \\ dX^m_{I,j}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a'_0 & -a'_1 & -a'_2 & & -a'_{m-1} \end{bmatrix} \begin{bmatrix} X^1_{I,j}(t) \\ X^2_{I,j}(t) \\ \vdots \\ X^m_{I,j}(t) \end{bmatrix} dt + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} dW^I_j(t), \qquad \begin{bmatrix} X^1_{I,j}(0) \\ X^2_{I,j}(0) \\ \vdots \\ X^m_{I,j}(0) \end{bmatrix} \quad (19)$$

$$I_j(t) = \begin{bmatrix} b'_0 & b'_1 \cdots b'_{m-1} \end{bmatrix} \begin{bmatrix} X^1_{I,j}(t) \\ X^2_{I,j}(t) \\ \vdots \\ X^m_{I,j}(t) \end{bmatrix} + D\,dW^I_j(t) + f^I_j(t)$$

the parameters $\{a'_{m-1},\ldots,a'_0, b'_{m-1},\ldots,b'_0, D\}$ are obtained from the approximation of the DPSD. The quadrature components have the same realization as in (19) except that subscript I is replaced with Q. Note that the parameters $\{a'_{m-1},\ldots,a'_0, b'_{m-1},\ldots,b'_0, D\}$ are in reality TV and have to be estimated on-line from the DPSD or time domain measurements. Equation (19) for the inphase and quadrature components of the jth path can be described as in (16) where

$$A_I = A_Q = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a'_0 & -a'_1 & -a'_2 & & -a'_{m-1} \end{bmatrix}, \; b_I = b_Q = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad (20)$$

$$C_I = C_Q = \begin{bmatrix} b'_0 & b'_1 \cdots b'_{m-1} \end{bmatrix}, \; D_I = D_Q = D$$

In the next subsection, the solution to (16) together with its statistics are derived. The statistics are shown to be TV.

C. Solution to the Stochastic State Space Models

The stochastic state space models described in (16) have a solution given by:

$$X_L(t) = e^{At}X_L(0) + \int_0^t e^{A(t-\tau)}\, B\, \dot{w}_L(\tau)\, d\tau \quad (21)$$

where $L = I$ or Q. Therefore, the mean of $X_L(t)$ is:

$$E\left[X_L(t)\right] = e^{At}\, E\left[X_L(0)\right] \quad (22)$$

and the covariance matrix of $X_L(t)$ is given by:

$$\begin{aligned} \Sigma_L(t) &= E\left[X_L(t) X_L^{tr}(t)\right] \\ &= e^{At} E\left[X_L(0) X_L^{tr}(0)\right] e^{A^{tr}t} + \int_0^t e^{A(t-\tau)} BB^{tr} e^{A^{tr}(t-\tau)} d\tau \end{aligned} \quad (23)$$

A simple differentiation of expression (23) shows that the covariance matrix $\Sigma_L(t)$ satisfies the Riccati equation:

$$\dot{\Sigma}_L(t) = A\Sigma_L(t) + \Sigma_L(t)A^{tr} + BB^{tr} \quad (24)$$

It can be seen in (22) and (23) that the mean and variance of the inphase and quadrature components are functions of time. Thus the statistics of the inphase and quadrature components, and therefore the statistics of the ad hoc channel, are times varying. Therefore, these stochastic state space models reflect the TV characteristics of the ad hoc channel.

The band-pass representation of the received signal corresponding to the jth path can be expressed as:

$$y(t) = \begin{bmatrix} \left(C_I X_I(t) + f_I(t)\right)\cos\omega_c t \\ -\left(C_Q X_Q(t) + f_Q(t)\right)\sin\omega_c t \end{bmatrix} s_l(t - \tau_j) \quad (25)$$

In the next section, link performance of ad hoc links using the developed stochastic models will be determined numerically and compared with the performance of cellular links.

VI. Link Performance for Ad Hoc Channels

Time-domain simulation of ad hoc channels can be performed by passing two independent white noise processes through two identical filters, obtained from the factorization of the DPSD, one for the in-phase and the other for the quadrature component [14], and realized in their state-space form as described in (16). The complex cepstrum approximation method is used to approximate the ad hoc DPSD with 4th order stable, minimum phase, real, and rational transfer function. The state space models developed in (20) is used for simulating the inphase and quadrature components of the ad hoc channel. The received signal is reproduced using (25). The simulation of the stochastic ad hoc channels is performed using Simulink in Matlab.

For simplicity we will consider the case of flat fading, in which the ad hoc channel has purely a multiplicative effect on the signal and the multipath components are not resolvable. Thus it can be considered as a single path [9]. BPSK is the assumed modulation technique and the carrier frequency is $f_c = 900\text{MHz}$. We test 10000 frames of P = 100 bits each. We focus on the single hop link and assume the mobile nodes are vehicles, with the constraint that the average speed over the mobile nodes is 30 km/hr. This implies $v_1 + v_2 = 60\,\text{km/hr}$, thus $f_{m_{1\to2}} = (v_1 + v_2) f_c / c = 50\,\text{Hz}$.

The cellular case is defined as the scenario where the link connects a mobile node with speed 30 km/hr to a permanently stationary node, which is the base station. Thus, there is only one mobile node, and the constraint is satisfied. The ad hoc case is when both nodes can move, and the average speed is 30 km/hr. We consider the NLOS case ($f_I = f_Q = 0$), which represents an environment with large obstructions. Figure (3) shows the attenuation coefficient for both the cellular case and the worst-case ad hoc case ($\alpha = 0$). It can be noticed from Figure (3) that an ad hoc link suffers from faster fading by noting the higher frequency components in the Rayleigh envelope of the worst-case ad hoc link. Also it can be noticed that deep fading (envelope less than –10 dB) on the ad hoc link occurs more frequently and less bursty. Therefore, the increased Doppler spread due to double mobility tends to smear the errors out, causing higher frame error rates.

Consider the data rate given by $R_b = P / T_c = 5$ Kbps which chosen such that the coherence time T_c equals the time it takes to send exactly one frame of length P bits, a condition where variation in Doppler spread greatly impacts the frame error rate (FER). Figure (4) shows the link performance for 10000 frames of 100 bits each. It is clear that the ad hoc link is worse than the cellular link, but the performance gap decreases as $\alpha \to 1$. This agrees with the main conclusion of [7], that an increase in degree of double mobility mitigates fading by lowering Doppler spread. The gain in performance is nonlinear with α, as the majority of gain is from $\alpha = 0$ to $\alpha = 0.5$. Intuitively, it seems to make sense that link performance improves as the degree of double mobility increases, since mobility in the network becomes distributed uniformly over the nodes in a kind of "equilibrium".

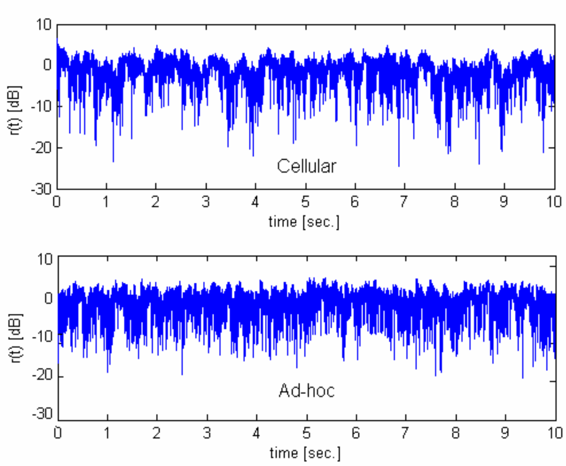

Fig. 3. Rayleigh attenuation coefficient for cellular link and worst-case ad hoc link.

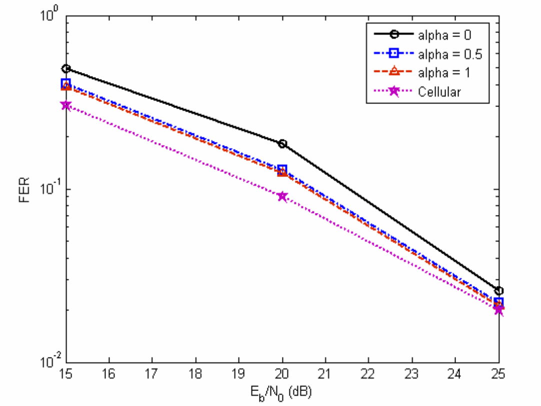

Fig. 4. FER results for Rayleigh flat ad hoc fading link for different α's compared with cellular link.

VII. Conclusion

Stochastic models for STF ad hoc wireless channels have been derived. These models take into account the statistical variations in ad hoc channels. The dynamics of the ad hoc channel is captured by a stochastic state space representation. The parameters of the stochastic state space models are determined by approximating the DPSD. The state space models have been used to simulate the effect of STF on the inphase and quadrature components of ad hoc networks. Numerical results show that for fixed $v_1 + v_2$, the higher α is, the more benign the small scale fading.

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