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Distributed Stochastic Power Control for Time-Varying Long-Term and Short-Term Fading Wireless Networks

Mohammed M. Olama, Seddik M. Djouadi, Charalambos D. Charalambous, and Samir Sahyoun

Abstract— **In this paper, new time-varying wireless channel models that capture both the space and time variations of long-term and short-term fading wireless networks are developed. The proposed models are based on stochastic differential equations. These models are more realistic than the static ones usually encountered in the literature. Moreover, optimal power control algorithms based on the new models are proposed. A centralized power control algorithm is shown to reduce to a simple linear programming problem if predictable power control strategies are used. In addition, an iterative distributed stochastic power control algorithm is used to solve for the optimization problem using stochastic approximations. The latter solely requires each mobile to know its received signal to interference ratio unlike common stochastic algorithms found in the literature. Numerical results show that the proposed distributed stochastic power control algorithm under the new time-varying channels provides better power stability and consumption than the deterministic ones.**

I. Introduction

POWER control (PC) is important to improve performance of wireless communication systems. The benefits of power minimization are not just increased battery life, but also increased overall network capacity. Users only need to expand sufficient power for acceptable reception as determined by their quality of service (QoS) specifications that is usually characterized by the signal to interference ratio (SIR) [1]. The majority of research papers in this field use time-invariant (static) wireless channel models for the PC problem. In time-invariant models, channel parameters are random but do not depend on time, and remain constant throughout the observation and estimation phase. This contrasts with time-varying (TV) models, where the channel dynamics become TV stochastic processes [2]-[4]. These models take into account the relative motion between transmitters and receivers and temporal variations of the propagating environment such as moving scatterers. They exhibit more realistic behavior of wireless networks.

In this paper, we consider dynamical (time-varying) modeling and PC of wireless channels. TV long-term fading (LTF) and short-term fading (STF) channel models are developed. The dynamics of wireless channels are captured by stochastic differential equations (SDEs). The SDE models proposed allow viewing the wireless channel as a dynamical system, which show how the channel evolves in time and space. In addition, they allow well-developed tools of adaptive and non-adaptive estimation and identification techniques (to estimate the model parameters) to be applied to this class of problems [5], [6].

Power control algorithms (PCAs) can be classified as centralized and distributed. The centralized PCAs require global out-of-cell information available at base stations. The distributed PCAs require base stations to know only the in-cell information, which can be easily obtained by local measurements. The power allocation problem has been studied extensively as an eigenvalue problem for non-negative matrices [1], [7], resulting in iterative PCAs that converge each user's power to the minimum power [8]-[10], and as optimization-based approaches [11]. Much of this previous work deals with static time-invariant channel models. In this paper, optimal PCAs based on the new TV models are proposed. The proposed centralized PCA is based on predictable power control strategies (PPCSs) that were first introduced in [2]. PPCSs simply mean updating the transmitted powers at discrete times and maintaining them fixed until the next power update begins. A distributed version of this algorithm is derived along the lines of [8].

Stochastic PCAs (SPCAs) that use noisy interference estimates have been introduced in [12], where conventional matched filter receivers are used. It is shown in [12] that the iterative stochastic PCA, which uses stochastic approximations, converges to the optimal power vector. These results were later extended to the cases when a nonlinear receiver or a decision feedback receiver is used [13]. However, in those papers the channel gains are assumed to be fixed, which ignore the effects of time-variations on the performance of the system. In this paper, the proposed distributed SPCA using stochastic approximations solely requires each mobile to know its received SIR. Numerical results are provided to evaluate the performance of the proposed PCAs.

The paper is organized as follows. In Section II, TV LTF and STF channel models in which the evolution of the channel is described by SDEs are introduced. In Section III, several PCAs are discussed. In Section III-A, a centralized deterministic PCA is proposed in which the solution is obtained through linear programming. A distributed SPCA is proposed in Section III-B. In Section IV, numerical results are presented. Finally, Section V provides the conclusion.

{Mohammed M. Olama, Seddik M. Djouadi, Samir Sahyoun} Department of Electrical and Computer Engineering, University of Tennessee, 1508 Middle Dr., Knoxville, TN 37996, USA (E-mail: molama@utk.edu, djouadi@ece.utk.edu, ssahyoun@utk.edu).

Charalambos D. Charalambous, Department of Electrical and Computer Engineering, University of Cyprus, 75, Kallipoleos Street, P.O.Box 20537, 1678 Nicosia, Cyprus (E-mail: chadcha@ucy.ac.cy).

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II. Time-Varying Multi-Path Fading Channel Models

Wireless channels can be classified as LTF and STF. In general, LTF and STF are considered as superimposed and may be treated separately [15]. In this paper, we consider TV modeling of both LTF and STF wireless channels. The proposed TV models that capture both types of fading are first introduced.

A. Stochastic LTF Wireless Channel Model

The *time-invariant* power loss (PL) in dB is [15]:

$$PL(d)[dB] := \overline{PL}(d_0)[dB] + 10\alpha \log\left(\frac{d}{d_0}\right) + \tilde{Z};\ \ d \ge d_0 \tag{1}$$

where $\overline{PL}(d_0)$ is the average PL in dB at a reference distance d_0 from the transmitter, α is the path loss exponent which depends on the propagation medium, and $\tilde{Z}$ is a zero-mean Gaussian distributed random variable, which represents the variability of the PL due to numerous reflections occurring along the path. In TV models, the PL becomes a random process denoted $\{X(t,\tau)\}_{t\ge 0, \tau \ge \tau_0}$, which is a function of both time t and location represented by τ, where $\tau = d/c$, d is the path length, c is the speed of light, $\tau_0 = d_0/c$ and d_0 is the reference distance. The process $\{X(t,\tau)\}_{t\ge 0, \tau \ge \tau_0}$ represents how much power the signal looses at a particular distance as a function of time. The signal attenuation is defined by $S(t,\tau) \triangleq e^{kX(t,\tau)}$, where $k = -\ln(10)/20$ [15].

The process $X(t,\tau)$ is generated by a mean-reverting version of a general linear SDE given by [16]:

$$\begin{aligned} dX(t,\tau) &= \beta(t,\tau)\big((\gamma(t,\tau) - X(t,\tau)\big)dt + \delta(t,\tau)dW(t), \\ X(t_0,\tau) &= N(\overline{PL}(d)[dB]; \sigma_{t_0}^2) \end{aligned} \tag{2}$$

where $\{W(t)\}_{t\ge 0}$ is a standard Brownian motion (zero drift, unit variance) which is assumed to be independent of $X(t_0,\tau)$, $N(\mu;\kappa)$ denotes a Gaussian random variable with mean μ and variance κ, and $\overline{PL}(d)[dB]$ is the average path loss in dB. The parameter $\gamma(t,\tau)$ models the *average* TV PL at distance d from transmitter, which corresponds to $\overline{PL}(d)[dB]$ at d indexed by t. This model tracks and converges to this value as time progresses. The instantaneous drift $\beta(t,\tau)\big(\gamma(t,\tau) - X(t,\tau)\big)$ represents the effect of pulling the process towards $\gamma(t,\tau)$, while $\beta(t,\tau)$ represents the speed of adjustment towards this value. Finally, $\delta(t,\tau)$ controls the instantaneous variance or volatility of the process for the instantaneous drift.

Define $\{\theta(t,\tau)\}_{t\ge 0} \triangleq \{\beta(t,\tau), \gamma(t,\tau), \delta(t,\tau)\}_{t\ge 0}$. If the random processes in $\{\theta(t,\tau)\}_{t\ge 0}$ are measurable and bounded, then (2) has a unique solution given by:

$$X(t,\tau) = e^{-\beta([t,t_0],\tau)} \cdot \left(X(t_0,\tau) + \int_{t_0}^{t} e^{\beta([u,t_0],\tau)} \big[\beta(u,\tau)\gamma(u,\tau)du + \delta(u,\tau)dW(u)\big] \right) \tag{3}$$

where $\beta([t,t_0],\tau) \triangleq \int_{t_0}^{t} \beta(u,\tau)du$. This model captures the temporal and spatial variations of the propagation environment as $\{\theta(t,\tau)\}_{t\ge 0}$ can be used to model the time and space varying characteristics of the channel.

In spatio-temporal lognormal model, the motion of mobiles is important factor to evaluate the TV PLs for the links involved. This can be illustrated in a simple way for the case of a single transmitter and a single receiver as shown in Fig. 1. At time t, the *average* PL at that new location is:

$$\gamma(t,\tau) = \overline{PL}(d(t))[dB] = \overline{PL}(d_0)[dB] + 10\alpha \log\frac{d(t)}{d_0} \tag{4}$$

where

$$d(t) = \sqrt{d^2 + (\upsilon t)^2 + 2dt\upsilon\cos\theta} \tag{5}$$

and $d(t) \ge d_0$. The parameter $\gamma(t,\tau)$ is used in the TV lognormal model (2) to obtain the TV PL $X(t,\tau)$.

Here we consider the uplink channel of a cellular network and we assume that users are already assigned to their base stations. Let M be the number of mobiles (users), and N be the number of base stations. The received signal of the ith mobile at its assigned base station at time t is given by:

$$y_i(t) = \sum_{j=1}^{M} \sqrt{p_j(t)}\, s_j(t) S_{ij}(t) + n_i(t) \tag{6}$$

where $p_j(t)$ is the transmitted power of mobile j at time t, which acts as a scaling on the information signal $s_j(t)$, $n_i(t)$ is the channel disturbance or noise at the base station of mobile i, and $S_{ij}(t)$ is the signal attenuation coefficient between mobile j and the base station assigned to mobile i. Therefore, in a cellular network the spatio-temporal model described in (2) for M mobiles and N base stations is:

$$\begin{aligned} dX_{ij}(t,\tau) &= \beta_{ij}(t,\tau)\big(\gamma_{ij}(t,\tau) - X_{ij}(t,\tau)\big)dt + \delta_{ij}(t,\tau)dW_{ij}(t), \\ X_{ij}(t_0,\tau) &= N\big(\overline{PL}(d)[dB]_{ij}; \sigma_{t_0}^2\big),\ \ 1 \le i, j \le M \end{aligned} \tag{7}$$

and the signal attenuation coefficients $S_{ij}(t,\tau)$ are generated using the relation $S_{ij}(t,\tau) = e^{kX_{ij}(t,\tau)}$, where $k = -\ln(10)/20$. The stochastic STF channel model is introduced next.

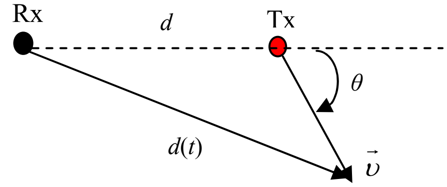

Fig. 1. A mobile (transmitter) at a distance d from a base station (receiver) moves with velocity ν and direction θ.

B. Stochastic STF Wireless Channel Model

The traditional STF model is based on Ossanna [17], and later expanded by Clarke [18], and Aulin [19]. Aulin's model is shown in Fig. 2. This model assumes that at each point between a transmitter and a receiver, the total received wave consists of superposition of N plane waves each having traveled via a different path. The nth wave is characterized by its field vector $E_n(t)$ given by:

$$E_n(t) = I_n(t)\cos\omega_c t - Q_n(t)\sin\omega_c t = \mathrm{Re}\{r_n(t)e^{j\Phi_n(t)}e^{j\omega_c t}\} \quad (8)$$

where $\{I_n(t), Q_n(t)\}$ are the corresponding inphase and quadrature components, $r_n(t) = \sqrt{I_n^2(t) + Q_n^2(t)}$ is the signal envelope, $\Phi_n(t) = \tan^{-1}(Q_n(t)/I_n(t))$ is the phase, ω_c is the carrier frequency, and {Re} denotes the real part. The total field $E(t)$ is given by:

$$E(t) = I(t)\cos\omega_c t - Q(t)\sin\omega_c t \quad (9)$$

where $\{I(t), Q(t)\}$ are inphase and quadrature components of the *total* wave with $I(t) = \sum_{n=1}^{N} I_n(t)$ and $Q(t) = \sum_{n=1}^{N} Q_n(t)$. The band-pass representation of the received signal, denoted by $y(t)$, is given by:

$$y(t) = \left[I(t)\cos\omega_c t - Q(t)\sin\omega_c t\right]s(t) \quad (10)$$

where $s(t)$ is the information signal.

According to Aulin's model, the DPSD is given by [19]:

$$S_D(f) = \begin{cases} 0, & |f| > f_m \\ \dfrac{E_0}{4 f_m \sin\beta_m}, & f_m\cos\beta_m \le |f| \le f_m \\ \dfrac{E_0}{4\pi f_m \sin\beta_m}\left[\dfrac{\pi}{2} - \sin^{-1}\left(\dfrac{2\cos^2\beta_m - 1 - (f/f_m)^2}{1 - (f/f_m)^2}\right)\right], & |f| < f_m\cos\beta_m \end{cases} \quad (11)$$

where β_m is the direction of the incident wave onto the receiver as illustrated in Fig. 2, f_m is the maximum Doppler frequency, and $E_0/2 = Var(I(t)) = Var(Q(t))$.

The main idea in constructing a TV STF channel model is to factorize the Doppler power spectral density (DPSD) into

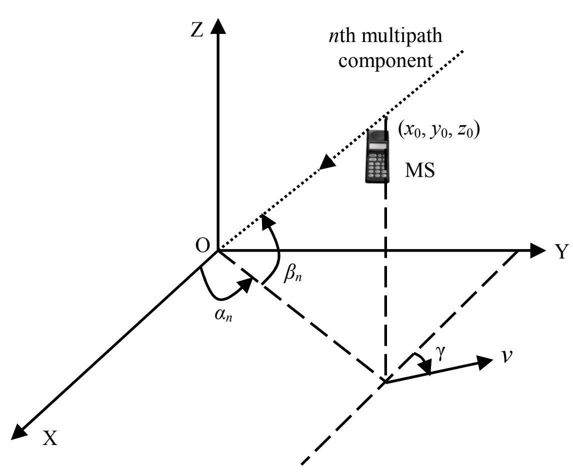

Fig. 2. Aulin's 3D multi-path scattering model

an approximate 4th order even transfer function, and then any stochastic realization can be used to obtain a state space representation for the inphase and quadrature components.

In order to approximate the power spectral density in (11), a 4th order even function in the form $\tilde{S}_D(s) = H(s)H(-s)$ is used with the factorization given by:

$$\tilde{S}_D(s) = \frac{K^2}{s^4 + 2\omega_n^2(1-2\zeta^2)s^2 + \omega_n^4}, H(s) = \frac{K}{s^2 + 2\zeta\omega_n + \omega_n^2} \quad (12)$$

where $\tilde{S}_D(s)$ is the approximation of $S_D(s)$. Expression (12) has three arbitrary parameters $\{\zeta, \omega_n, K\}$, which can be adjusted such that the approximate curve coincides with the actual curve at different points. In fact, if these parameters are chosen such that:

$$\zeta = \sqrt{\frac{1}{2}\left(1 - \sqrt{1 - \frac{S_D(0)}{S_D(j\omega_{\max})}}\right)}, \omega_n = \frac{\omega_{\max}}{\sqrt{1-2\zeta^2}}, K = \omega_n^2\sqrt{S_D(0)} \quad (13)$$

then the approximate density $\tilde{S}_D(s)$ coincides with the exact density $S_D(s)$ at $\omega = 0$ and $\omega = \omega_{\max}$.

The SDE, which corresponds to $H(s)$ in (12) is given by:

$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = K\dot{w}(t) \quad (14)$$

where $\dot{x}(0), x(0)$ are given, $\{\dot{w}(t)\}_{t\ge 0}$ is a white-noise process. Expression (14) can be written in terms of inphase and quadrature components as:

$$\begin{aligned} &\ddot{x}_I(t) + 2\zeta\omega_n\dot{x}_I(t) + \omega_n^2 x_I(t) = K\dot{w}_I(t) \\ &\ddot{x}_Q(t) + 2\zeta\omega_n\dot{x}_Q(t) + \omega_n^2 x_Q(t) = K\dot{w}_Q(t) \end{aligned} \quad (15)$$

where $\{\dot{w}_I(t)\}_{t\ge 0}$ and $\{\dot{w}_Q(t)\}_{t\ge 0}$ are two independent and identically distributed (*i.i.d.*) white Gaussian noises. Equation (15) can be realized in a stochastic state-space controllable canonical form as:

$$\begin{aligned} &\dot{X}_I(t) = A_I X_I(t) + B_I\dot{w}_I(t), \quad X_I(0) \in \Re^2; \\ &I(t) = C_I X_I(t) + f^I(t), \\ &\dot{X}_Q(t) = A_Q X_Q(t) + B_Q\dot{w}_Q(t), \quad X_Q(0) \in \Re^2, \\ &Q(t) = C_Q X_Q(t) + f^Q(t), \end{aligned} \quad (16)$$

where $X_I(t) = \begin{bmatrix} x_I(t) \\ \dot{x}_I(t) \end{bmatrix}$, $X_Q(t) = \begin{bmatrix} x_Q(t) \\ \dot{x}_Q(t) \end{bmatrix}$, $B_I = B_Q = \begin{bmatrix} 0 \\ K \end{bmatrix}$

$A_I = A_Q = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix}$, $C_I = C_Q = [1 \quad 0]$, $f^I(t)$ and $f^Q(t)$ are arbitrary functions representing the line of sight (LOS) of the inphase and quadrature components respectively, and $\Re^2$ denotes the real two-dimensional Euclidean space. The parameters $\{\zeta, \omega_n, K\}$ are obtained from approximating the DPSD as described in (13). Define,

$$X(t) := \begin{bmatrix} X_I(t) \\ X_Q(t) \end{bmatrix}, A := \begin{bmatrix} A_I & 0 \\ 0 & A_Q \end{bmatrix}, B := \begin{bmatrix} B_I & 0 \\ 0 & B_Q \end{bmatrix}, \dot{w}(t) := \begin{bmatrix} \dot{w}_I(t) \\ \dot{w}_Q(t) \end{bmatrix} \quad (17)$$

then the state equations in (16) can be described as:

$$\dot{X}(t) = AX(t) + B\dot{w}(t), \quad X(0) \in \Re^4 \quad (18)$$

and the received signal can be expressed as:

$$y(t)=\begin{bmatrix}\left(C_I\,X_I(t)+f^I(t)\right)\cos\omega_c t\\ -\left(C_Q\,X_Q(t)+f^Q(t)\right)\sin\omega_c t\end{bmatrix}s(t) \tag{19}$$

Now consider a cellular network with M mobiles and N base stations. The received signal of the ith mobile at its assigned base station at time t can be expressed as:

$$y_i(t)=\sum_{j=1}^{M}\sqrt{p_j(t)}s_j(t)\left(I_{ij}(t)\cos\omega_c t-Q_{ij}(t)\sin\omega_c t\right)+n_i(t) \tag{20}$$

where processes $I_{ij}(t)$ and $Q_{ij}(t)$ are the channel inphase and quadrature components between mobile j and the base station assigned to mobile i, respectively. Letting $H_{nj}(t)\triangleq\left[\cos\omega_c t \quad 0 \quad -\sin\omega_c t \quad 0\right]s_j(t)$, $X_{ij}(t)\triangleq\begin{bmatrix}X_{I_{ij}}(t)\\ X_{Q_{ij}}(t)\end{bmatrix}$, and $f_{ij}(t)=\left[f_{ij}^I(t) \quad 0 \quad f_{ij}^Q(t) \quad 0\right]^{tr}$. Then the state space representation of a STF wireless network can be written as:

$$\begin{aligned}dX_{ij}(t)&=A_{ij}X_{ij}(t)dt+B_{ij}dw_{ij}(t)\\ y_i(t)&=\sum_{k=1}^{M}\sqrt{p_k(t)}s_k(t)H_{ik}(t)\left[X_{ik}(t)+f_{ik}(t)\right]+n_i(t)\end{aligned} \tag{21}$$

where $1\le i,j\le M$. The TV LTF and STF channel models in (7) and (21) are used to generate the link gains of wireless networks for the PCAs proposed in the next section.

III. Power Control Algorithms In Time-Varying Wireless Netwoks

A. Deterministic PC Schemes

Consider the cellular network described in the previous section, the centralized PC problem for *time-invariant* channels can be stated as follows [2]:

$$\min_{(p_1\ge 0,\ldots,p_M\ge 0)}\sum_{i=1}^{M}p_i \quad \text{subject to} \quad \frac{p_i g_{ii}}{\sum_{j\ne i}^{M}p_j g_{ij}+\eta_i}\ge\varepsilon_i \tag{22}$$

where $1\le i\le M$, p_i is the power of mobile i, $g_{ij}>0$ is the *time-invariant* channel gain between mobile j and the base station assigned to mobile i, $\varepsilon_i>0$ is the target SIR of mobile i, and $\eta_i>0$ is the noise power level at the base station of mobile i. Expression (22) for the TV LTF and STF channel models in (7) and (21) respectively, described using path-wise QoS of each user over a time interval $[0,T]$ is:

$$\min_{(p_1\ge 0,\ldots,p_M\ge 0)}\left\{\sum_{i=1}^{M}\int_0^T p_i(t)dt\right\},\quad \text{subject to}$$

$$\frac{\int_0^T p_i(t)s_i^2(t)S_{ii}^2(t)dt}{\sum_{k\ne i}^{M}\int_0^T p_k(t)s_k^2(t)S_{ik}^2(t)dt+\int_0^T n_i^2(t)dt}\ge\varepsilon_i,\quad \text{for LTF} \tag{23}$$

$$\frac{\int_0^T p_i(t)s_i^2(t)\left[H_{ii}(t)X_{ii}(t)\right]^2dt}{\sum_{k\ne i}^{M}\int_0^T p_k(t)s_k^2(t)\left[H_{ik}(t)X_{ik}(t)\right]^2dt+\int_0^T n_i^2(t)dt}\ge\varepsilon_i,\text{for STF}$$

and $i=1,\cdots,M$. A solution to (23) is presented by using the concept of predictable power control strategies (PPCS) [2]. Consider a set of discrete time strategies $\left\{p_i(t_k)\right\}_{i=1}^{M}$, $0=t_0<t_1<\ldots<t_k<t_{k+1}<\ldots\le T$. At time t_{k-1}, the base stations observe or estimate the channel information $\left\{S_{ij}(t_{k-1}),s_i(t_{k-1})\right\}_{i,j=1}^{M}$ for LTF or $\left\{I_{ij}(t_{k-1}),Q_{ij}(t_{k-1}),s_i(t_{k-1})\right\}_{i,j=1}^{M}$ for STF. Using the concept of predictable strategy, the base stations determine the control strategy $\left\{p_i(t_k)\right\}_{i=1}^{M}$ for the next time instant t_k. The latter is communicated back to the mobiles, which hold these values during the time interval $\left[t_{k-1},t_k\right)$. At time t_k, a new set of channel information $\left\{S_{ij}(t_k),s_i(t_k)\right\}_{i,j=1}^{M}$ or $\left\{I_{ij}(t_k),Q_{ij}(t_k),s_i(t_k)\right\}_{i,j=1}^{M}$ is observed at the base stations and the time t_{k+1} control strategies $\left\{p_i(t_{k+1})\right\}_{i=1}^{M}$ are computed and communicated back to the mobiles which hold them constant during the time interval $\left[t_k,t_{k+1}\right)$. Such decision strategies are called predictable. Using the concept of PPCS over any time interval $\left[t_k,\,t_{k+1}\right]$, equation (23) is equivalent to:

$$\begin{aligned}&\min_{\mathbf{p}(t_{k+1})>0}\sum_{i=1}^{M}p_i\left(t_{k+1}\right)\quad \text{subject to}\\ &\mathbf{p}\left(t_{k+1}\right)\ge\mathbf{\Gamma}\mathbf{G}_I^{-1}\left(t_k,t_{k+1}\right)\times\left(\mathbf{G}\left(t_k,t_{k+1}\right)\mathbf{p}\left(t_{k+1}\right)+\boldsymbol{\eta}\left(t_{k+1}\right)\right)\end{aligned} \tag{24}$$

where

$$g_{ij}\left(t_k,t_{k+1}\right):=\int_{t_k}^{t_{k+1}}s_j^2(t)S_{ij}^2(t)dt,\ 1\le i,\ j\le M \quad \text{for LTF,}$$

$$:=\int_{t_k}^{t_{k+1}}s_j^2(t)\left[H_{ij}(t)X_{ij}(t)\right]^2dt,\ 1\le i,\ j\le M \quad \text{for STF,}$$

$$\mathbf{p}\left(t_{k+1}\right):=\left(p_1\left(t_{k+1}\right),\cdots,p_M\left(t_{k+1}\right)\right)^{tr},\ \eta_i\left(t_k,t_{k+1}\right):=\int_{t_k}^{t_{k+1}}n_i^2(t)dt,$$

$$\mathbf{G}_I\left(t_k,t_{k+1}\right):=diag\left(g_{11}\left(t_k,t_{k+1}\right),\cdots,g_{MM}\left(t_k,t_{k+1}\right)\right),$$

$$\mathbf{G}\left(t_k,t_{k+1}\right):=\left\{\begin{matrix}0 & ,\text{ if } i=j\\ g_{ij}\left(t_k,t_{k+1}\right) & ,\text{ if } i\ne j\end{matrix}\right\},1\le i,\ j\le M$$

$$\boldsymbol{\eta}\left(t_k,t_{k+1}\right):=\left(\eta_1\left(t_k,t_{k+1}\right),\cdots,\eta_M\left(t_k,t_{k+1}\right)\right)^{tr},\quad \mathbf{\Gamma}:=diag\left(\varepsilon_1,\cdots,\varepsilon_M\right),$$

$diag(\cdot)$ denotes a diagonal matrix with its argument as diagonal entries, and "^{tr}" stands for matrix or vector transpose. The optimization in (24) is a linear programming problem in $M\times 1$ vector of unknowns $\mathbf{p}\left(t_{k+1}\right)$. Here $\left[t_k,\,t_{k+1}\right]$ is a time interval such that the channel model does not change significantly, i.e., $\left[t_k,\,t_{k+1}\right]$ should be smaller than the coherence time of the channel.

Next, we consider an iterative distributed version of the centralized PCA in (24). This is convenient for on-line implementation since it helps autonomous execution at the node or link level, requiring minimal usage of network communication resources for control signaling. The iterative distributed PCA proposed in [8] and [10] can be used to find a distributed version to the centralized PCA in (24). The constraint in (24) can be rewritten as:

$$\left(\mathbf{I}-\mathbf{\Gamma G}_I^{-1}(t_k,t_{k+1})\mathbf{G}(t_k,t_{k+1})\right)\mathbf{p}(t_{k+1})\geq \mathbf{\Gamma G}_I^{-1}(t_k,t_{k+1})\boldsymbol{\eta}(t_{k+1}) \quad (25)$$

Defining $\mathbf{F}(t_k,t_{k+1}) \triangleq \mathbf{\Gamma G}_I^{-1}(t_k,t_{k+1})\mathbf{G}(t_k,t_{k+1})$ and $\mathbf{u}(t_k,t_{k+1}) \triangleq \mathbf{\Gamma}\ \mathbf{G}_I^{-1}(t_k,t_{k+1})\ \boldsymbol{\eta}(t_{k+1})$, then (25) becomes:

$$\left(\mathbf{I}-\mathbf{F}(t_k,t_{k+1})\right)\mathbf{p}(t_{k+1}) \geq \mathbf{u}(t_k,t_{k+1}) \quad (26)$$

If the channel gains are *time-invariant*, i.e., $\mathbf{F}(t_k,t_{k+1})=\mathbf{F}$ and $\mathbf{u}(t_k,t_{k+1})=\mathbf{u}$, then the power control problem is feasible if $\rho_{\mathbf{F}} < 1$, where $\rho_{\mathbf{F}}$ is the Perron-Frobenius eigenvalue of $\mathbf{F}$ [8]. It is shown in [8] and [10] that the following iterative PCA converges to the minimal power vector when $\rho_{\mathbf{F}} < 1$:

$$\mathbf{p}(t_{k+1})=\mathbf{F}\mathbf{p}(t_k)+\mathbf{u} \quad (27)$$

However, our channel gains are *time-varying*, thus a "time-varying version" of the deterministic PCA (DPCA) in (27) can be defined as:

$$\mathbf{p}(t_{k+1})=\mathbf{F}(t_k,t_{k+1})\mathbf{p}(t_k)+\mathbf{u}(t_k,t_{k+1}) \quad (28)$$

Since $\mathbf{F}(t_k,t_{k+1})$ is a random matrix-valued process, the key convergence condition is that the Lyapunov exponent $\lambda_{\mathbf{F}} < 0$ [20], where $\lambda_{\mathbf{F}}$ is defined as:

$$\lambda_{\mathbf{F}}=\lim_{k\to\infty}\frac{1}{k}\log\left\|\mathbf{F}(t_0,t_1)\mathbf{F}(t_1,t_2)...\mathbf{F}(t_k,t_{k+1})\right\| \quad (29)$$

Throughout this section, we assume that the PC problem is feasible, i.e., there exists a power vector $\mathbf{p}(t_k)$ that satisfies the inequality in (24) for all t_k in [0,*T*]. The distributed version of (28) can be written as:

$$p_i(t_{k+1})=\frac{\varepsilon_i(t_k)}{R_i(t_k)}p_i(t_k),\ i=1,...,M \quad (30)$$

where $R_i(t_k)$ is the instantaneous SIR defined by:

$$R_i(t_k)=\frac{p_i(t_k)g_{ii}(t_k,t_{k+1})}{\sum_{j\neq i}^{M}p_j(t_k)g_{ij}(t_k,t_{k+1})+\eta_i(t_k,t_{k+1})}, \quad (31)$$

It is shown in [14] that the performance of the DPCA in (30) in terms of power consumption is not optimal when the channel environment is TV. Actually, the performance can be severely degraded when PCAs that are designed for deterministic channels are applied to TV channels [14]. Therefore, stochastic PCAs (SPCAs) must be used in order to ensure stable optimal power consumption. The latter is introduced in the following section.

B. Stochastic PC Schemes

A distributed SPCA similar to the one described in [12] is used in this section, where the transmit powers are updated based on stochastic approximations. Let us define the instantaneous interference at time t_k by:

$$I_i(t_k)=\sum_{j\neq i}^{M}p_i(t_k)g_{ij}(t_k,t_{k+1})+\eta_i(t_k,t_{k+1}),\ i=1,...,M \quad (32)$$

then the SPCA proposed in [12], which uses the concept of interference averaging, can be used to update the transmitted power recursively as:

$$p_i(t_{k+1})=\left(1-a(t_k)\right)p_i(t_k)+a(t_k)\frac{\varepsilon_i(t_k)}{g_{ii}(t_k,t_{k+1})}\left[I_i(t_k)\right] \quad (33)$$

where $a(t_k)$ is the step-size at time t_k. Substituting (32) into (33) and using (31) yields:

$$p_i(t_{k+1})=\left(1-a(t_k)\right)p_i(t_k)+a(t_k)\frac{\varepsilon_i(t_k)}{R_i(t_k)}p_i(t_k) \quad (34)$$

If the PC problem in (24) is feasible, the distributed SPCA in (34) converges in probability to the optimal power vector when the step-size sequence satisfies $\sum_{k=0}^{\infty}a(t_k)=\infty$ [21]. In fact, the error between the power vector and the optimal value does not vanish for non-vanishing step-size sequence; this is the price paid in order to make the algorithm in (34) able to track TV environments. This algorithm is fully distributed in the sense that each user iteratively updates its power level by estimating the received SIR of its own channel. It does not require any knowledge of the link gains and state information of other users. It is worth mentioning that the proposed distributed SPCA in (34) is different from the algorithm proposed in [14] where two parameters, namely, the received SIRs $R_i(t_k)$ and the channel gains $g_{ii}(t_k,t_{k+1})$, are required to be known. However, only $R_i(t_k)$ are required in (34).

IV. NUMERICAL EXAMPLES

In this section, we give two numerical examples to determine the performance of the proposed SPCA under the developed TV LTF and STF wireless networks.

Example 1. TV LTF Wireless Network:

The LTF cellular network has the following features: The number of transmitters (mobiles) is M = 24, the information signal $s_i(t)=1$ for $i=1,...,M$, target SIR ε_i = 5 for $i=1,...,M$, the step-size sequence is $a(t_k)=0.1$, the transmitted powers are computed every 15 milliseconds, i.e., $[t_k,\ t_{k+1}]=15$ milliseconds. The simulation is performed for 1.5 seconds, initial distances of all mobiles with respect to their own base stations d_{ii} are generated as uniformly independent identically distributed (*i.i.d.*) random variables (*r.v.*'s) in [10 – 100] meters, cross initial distances of all mobiles with respect to other base stations d_{ij}, $i\neq j$, are generated as uniformly *i.i.d. r.v.*'s in [250 - 550] meters, the angle θ_{ij} between the direction of motion of mobile *j* and the distance vector passes through base station i and the mobile j are generated as uniformly *i.i.d. r.v.*'s in [0 – 180] degrees, the average velocities of mobiles are generated as uniformly *i.i.d. r.v.*'s in [40 – 100] km/hr, PL exponent is 3.5, initial reference distance from each of the transmitters is 10 m, PL at the initial reference distance is 15 dB, $\delta(t) = 1400$ and $\beta(t) = 225000$ for the SDE's, η_i's for $i = 1,...,M$ are *i.i.d.* Gaussian *r.v.*'s with zero mean and variance = 10^{-12} W.

Example 2. TV STF Wireless Network:

The STF cellular network has the following features: Number of mobiles M, number of base stations N, target SIRs ε_i, step-size sequence $a(t_k)$, $[t_k, t_{k+1}]$, $[0, T]$, average velocities of mobiles and η_i's are the same as in Example 1, f_c = 910 MHz, E_{0ii}'s are uniformly *i.i.d. r.v.*'s in the range [400-600], E_{0ij}'s ($i \neq j$) are uniformly *i.i.d. r.v.*'s in [25-150], angles of arrival $\beta_{m_{ij}}$'s for each link are generated as uniformly *i.i.d. r.v.*'s in [0 – 16] degrees.

Fig. 3 and Fig. 4 show the total transmitted power of all mobiles using the distributed DPCA in (30) and the SPCA in (34) under stochastic TV LTF and STF wireless network described in Example 1 and Example 2, respectively. Note that the power axis is *logarithmic*. Clearly, the distributed SPCA using stochastic approximations provides better power stability and consumption than that of the DPCA.

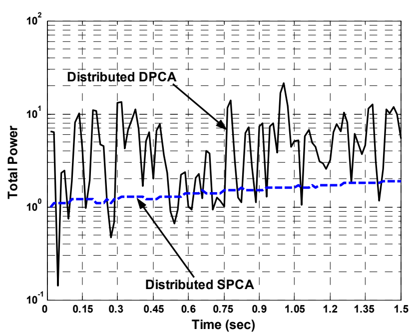

Fig. 3. Sum of transmitted power of all mobiles for the distributed DPCA and SPCA under TV LTF channels.

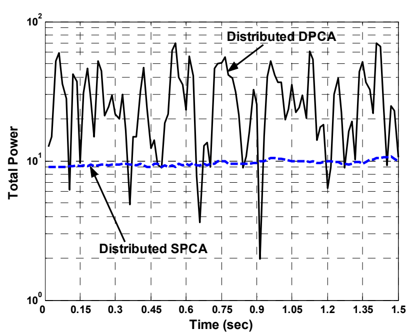

Fig. 4. Sum of transmitted power of all mobiles for the distributed DPCA and SPCA under TV STF channels.

V. Conclusion

In this paper, TV LTF and STF wireless channel models, which capture both the space and time variations of wireless channels, are developed. The proposed models are more realistic than the static models encountered in the literature. Moreover, optimal PCAs based on the developed models are proposed. The optimal DPCA is shown to reduce to a simple linear programming problem if PPCS are used. In addition, an iterative distributed SPCA is used to solve for the optimization problem using stochastic approximations. The latter solely requires each mobile to know its received SIR unlike common SPCAs found in the literature.

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