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Time Varying Wireless Channel Modeling, Estimation, Identification, and Power Control from Measurements

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Abstract—**This paper is concerned with a time-varying wireless channel modeling, its parameter estimation, system identification, and optimal power control from measurement data. The channel model is represented in state space form, while the Expectation Maximization algorithm and Kalman filtering are used in the channel parameter and state estimation, respectively. The inphase and quadrature components of the wireless channel and its parameters are estimated from received signal measurements. The proposed algorithm is tested using measurement data, and the results are presented. Moreover, an optimal power control algorithm based on the estimated parameters and channel states is proposed. Numerical results indicate that a significant gain in performance can be achieved using the proposed algorithms.**

I. Introduction

Time varying (TV) wireless channel models capture both the space and time variations of wireless systems, which are due to the relative mobility of the receiver, transmitter and/or scatterers [1]-[3]. This paper is concerned with the development of TV wireless channel models based on system identification algorithms to extract various channel parameters using received signal measurement data.

The majority of research papers in this field such as in [4]-[6] use time-invariant (static) models for wireless channels. In time-invariant models, channel parameters are random but do not depend on time, and remain constant throughout the observation and estimation phase. This contrasts with TV models, where the channel dynamics become TV stochastic processes [1]-[3].

In [1] and [3], the TV channel parameters are estimated from approximating the Doppler power spectral density (DPSD) of the wireless fading channel. However, in reality one can not have access to the TV DPSD at all times during the estimation process. We propose to estimate channel parameters as well as the inphase and quadrature components directly from the received signal measurements, which are usually available or easy to obtain in any wireless network. The Expectation Maximization (EM) algorithm and Kalman filtering are employed in the estimation process.

The developed TV channel models from received signal level measurements are useful in most wireless applications. These models are used to develop an optimal power control algorithm (PCA). Power control (PC) is important to improve performance of wireless communication systems. The benefits of power minimization are not just increased battery life, but also increased overall network capacity. The power allocation problem has been studied extensively as an eigenvalue problem for non-negative matrices [7], [8], resulting in iterative PCAs that converge each user's power to the minimum power [9], [10], and as optimization-based approaches [11]. Much of this previous work deals with static time-invariant channel models.

The proposed PCA is based on predictable power control strategies (PPCS) that were first introduced in [1]. PPCS simply means updating the transmitted powers at discrete times and maintaining them fixed until the next power update begins. The PPCS algorithm is proven to be effectively applicable to such dynamical models for an optimal PC. The outage probability (OP) is used as a performance measure. Since few TV dynamical channel models have so far been investigated with the application of any PCA, the suggested dynamical models and PCA will thus provide a far more realistic and efficient optimal control of wireless networks.

The paper is organized as follows. In Section II, the TV wireless fading channel mathematical model is introduced. In Section III, the EM algorithm together with the Kalman filter, to estimate the channel parameters as well as the inphase and quadrature components from received signal measurements, is developed. In Section IV, a PCA based on the developed channel models is discussed. In Section V, numerical results are presented. Finally, Section VI provides the conclusion.

II. Mathematical Models For Wireless Channels

The general TV model of a wireless channel is typically represented by the band-pass impulse response [5]:

$$C(t;\tau)=\sum_{j=1}^{J(t)}\Big(I_j(t,\tau)\cos(\omega_c t)-Q_j(t,\tau)\sin(\omega_c t)\Big)\delta\big(\tau-\tau_j(t)\big) \quad (1)$$

where $C(t;\tau)$ is the band-pass response of the channel at time t, due to an impulse applied at time $t-\tau$, $J(t)$ is the random number of multipath components, ω_c is the carrier frequency, and the set $\{I_j(t,\tau),Q_j(t,\tau),\tau_j(t)\}_{j=1}^{J(t)}$ describes the random TV inphase component, quadrature component, and arrival time of the different paths, respectively. Let

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$s_l(t)$ be the low pass equivalent representation of the transmitted signal, then the band pass representation of the received signal is given by:

$$y(t)=\sum_{j=1}^{J(t)}\Big(I_j(t,\tau)\cos(\omega_c t)-Q_j(t,\tau)\sin(\omega_c t)\Big)s_l\big(t-\tau_j(t)\big) + v_I(t)\cos(\omega_c t)-v_Q(t)\sin(\omega_c t) \tag{2}$$

where $\{v_I(t)\}_{t\geq 0}$ and $\{v_Q(t)\}_{t\geq 0}$ are two independent and identically distributed (iid) white Gaussian noise processes, with density $\mathcal{N}(0;\sigma_v^2)$.

It is shown in [1], [3], and [12] that the DPSD of the wireless fading channel, denoted by $S(s)$, can be approximated by an even, stable, rational, and factorizable transfer function, $\tilde{S}(s)=H(s)H(-s)$, where $H(s)$ is:

$$H(s)=\frac{b'_{n-1}s^{n-1}+...+b'_1 s+b'_0}{s^n+a'_{n-1}s^{n-1}+...+a'_1 s+a'_0} \tag{3}$$

Consequently, the inphase and quadrature components can be realized using the following stochastic observable canonical form (OCC) state space representation:

$$\begin{aligned} dX_{I,j}(t)&=A_I X_{I,j}(t)dt+B_I dW_j^I(t)\\ I_j(t)&=C_I X_{I,j}(t)+f_j^I(t)\\ dX_{Q,j}(t)&=A_Q X_{Q,j}(t)dt+B_Q dW_j^Q(t)\\ Q_j(t)&=C_Q X_{Q,j}(t)+f_j^Q(t) \end{aligned} \tag{4}$$

where

$$\begin{aligned} &X_{I,j}(t)=\left[X_{I,j}^1(t),X_{I,j}^2(t),\dots,X_{I,j}^n(t)\right]^T,\\ &X_{Q,j}(t)=\left[X_{Q,j}^1(t),X_{Q,j}^2(t),\dots,X_{Q,j}^n(t)\right]^T,\\ &A_I=A_Q=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & 1\\ -a'_0 & -a'_1 & -a'_2 & & -a'_{n-1}\end{bmatrix}, B_I=B_Q=\begin{bmatrix} b'_{n-1}\\ \vdots\\ \vdots\\ b'_1\\ b'_0\end{bmatrix},\\ &C_I=C_Q=\left[1\ \ 0\cdots 0\right], \end{aligned} \tag{5}$$

and $\left\{W_j^I(t)\right\}_{t\geq 0}$, $\left\{W_j^Q(t)\right\}_{t\geq 0}$ are independent standard Brownian motions, which correspond to the inphase and quadrature components of the nth path respectively, $f_j^I(t)$ and $f_j^Q(t)$ are arbitrary functions representing the line-of-sight (LOS) of the inphase and quadrature components respectively. Without loss of generality, we consider the case of flat fading, in which the fading channel has purely a multiplicative effect on the signal and the multipath components are not resolvable. Thus, it can be considered as a single path [5]. We also consider the non-line-of-sight (NLOS) case, i.e., $f_j^I(t)=f_j^Q(t)=0$, which represents an environment with large obstructions.

Similarly, following the state space representation in (4), the fading channel can be represented using general stochastic state space representation of the form:

$$\begin{aligned} \dot{X}(t)&=AX(t)+Bw(t)\\ y(t)&=CX(t)+Dv(t) \end{aligned} \tag{6}$$

where

$$\begin{aligned} &X(t)=\left[X_I(t)\quad X_Q(t)\right]^T, A=\begin{bmatrix} A_I & 0\\ 0 & A_Q\end{bmatrix}, B=\begin{bmatrix} B_I & 0\\ 0 & B_Q\end{bmatrix},\\ &C=\left[\cos(\omega_c t)C_I\quad -\sin(\omega_c t)C_Q\right], v(t)=\left[v_I(t)\quad v_Q(t)\right]^T\\ &w(t)=\left[dW^I(t)\quad dW^Q(t)\right]^T, D=\left[\cos(\omega_c t)\quad -\sin(\omega_c t)\right], \end{aligned} \tag{7}$$

In this case, $y(t)$ represents the received signal measurements, $X(t)$ is the state variable of the inphase and quadrature components, $w(t)$ and $v(t)$ are the process and measurement noises, respectively, which are assumed independent Gaussian.

In [1], [3], and [12], the channel parameters $\left\{a'_{n-1},...,a'_0,b'_{n-1},...,b'_0\right\}$ are obtained from approximating the DPSD. However, in reality one can not have access to the DPSD at all times during the estimation process. Therefore, in this paper channel parameters as well as inphase and quadrature components are estimated directly from received signal measurements, which are usually available or easy to obtain in any wireless network. The EM algorithm and Kalman filtering are employed in the channel parameter and state estimation, respectively. These algorithms are introduced in the next section.

III. Wireless Channel Estimation Via the Expectation Maximization and Kalman Filtering

This section describes the procedure employed to estimate the channel model parameters and states associated with the state space model in (6), using the EM algorithm [13] together with Kalman filtering [14]. However, for simplicity we consider the discrete-time version of (6) given by:

$$\begin{aligned} x_{t+1}&=A_t x_t+B_t w_t\\ y_t&=C_t x_t+D_t v_t \end{aligned} \tag{8}$$

where $x_t\in\Re^n$ is a state vector, $y_t\in\Re^d$ is a measurement vector, $w_t\in\Re^m$ is a state noise, and $v_t\in\Re^d$ is a measurement noise. The noise processes w_t and v_t are assumed to be independent zero mean and unit variance Gaussian processes.

The unknown system parameters $\theta_t=\{A_t,B_t,C_t,D_t\}$ as well as the system states x_t are estimated through the received signal measurement data, $Y_N=\{y_1,y_2,...,y_N\}$. The methodology employed is recursive and based on the EM algorithm together with the Kalman filter. The Kalman filter is introduced next.

A. Channel State Estimation: The Kalman Filter

The Kalman filter estimates the channel states x_t for given system parameter θ_t and measurements Y_t. It is described by the following equations [14]:

$$\begin{aligned}\hat{x}_{t|t} &= A_t\hat{x}_{t-1|t-1} + P_{t|t}C_t^T D_t^{-2}\left(y_t - C_t A_t \hat{x}_{t-1|t-1}\right)\\ \hat{x}_{t|t-1} &= A_t\hat{x}_{t-1|t-1},\ \hat{x}_{0|0} = m_0\end{aligned} \quad (9)$$

where $t = 0,1,2,...,N$, and $P_{t|t}$ is given by:

$$\begin{aligned}\overline{P}_{t|t}^{-1} &= P_{t-1|t-1}^{-1} + A_t^T B_t^{-2} A_t\\ P_{t|t}^{-1} &= C_t^T D_t^{-2} C_t + B_t^{-2} - B_t^{-2}\overline{P}_{t|t}A_t^T B_t^{-2}\\ P_{t|t-1} &= A_t P_{t-1|t-1}A_t^T + B_t^2\end{aligned} \quad (10)$$

The channel parameters $\theta_t = \{A_t, B_t, C_t, D_t\}$ are estimated using the EM algorithm which is introduced next.

B. Channel Parameter Estimation: The EM Algorithm

The EM algorithm uses a bank of Kalman filters to yield a maximum likelihood (ML) parameter estimate of the Gaussian state space model. The EM algorithm is an iterative scheme for computing the ML estimate of the system parameters θ_t, given the data Y_t. Specifically, each iteration of the EM algorithm consists of two steps: The expectation step and the maximization step. The EM algorithm is described by [13]:

$$\begin{aligned}\hat{A}_t &= E\left(\sum_{k=1}^{t} x_k x_{k-1}^T \mid Y_t\right)\times\left[E\left(\sum_{k=1}^{t} x_k x_k^T \mid Y_t\right)\right]^{-1}\\ \hat{B}_t^2 &= \frac{1}{t}E\left(\sum_{k=1}^{t}\left(\left(x_k - A_k x_{k-1}\right)\left(x_k - A_k x_{k-1}\right)^T\right)\mid Y_t\right)\\ &= \frac{1}{t}E\left(\sum_{k=1}^{t}\begin{pmatrix}\left(x_k x_k^T\right) - A_k\left(x_k x_{k-1}^T\right)^T\\ -\left(x_k x_{k-1}^T\right)A_k^T + A_k\left(x_{k-1}x_{k-1}^T\right)A_k^T\end{pmatrix}\mid Y_t\right)\\ \hat{C} &= E\left(\sum_{k=1}^{t} y_k x_k^T \mid Y_t\right)\times\left[E\left(\sum_{k=1}^{t} x_k x_k^T \mid Y_t\right)\right]^{-1}\\ \hat{D}_t^2 &= \frac{1}{t}E\left(\sum_{k=1}^{t}\left(\left(y_k - C_k x_k\right)\left(y_k - C_k x_k\right)^T\right)\mid Y_t\right)\\ &= \frac{1}{t}E\left(\sum_{k=1}^{t}\begin{pmatrix}\left(y_k y_k^T\right) - A_k\left(y_k x_k^T\right)C_k^T\\ -C_k\left(y_k x_k^T\right)^T + C_k\left(x_k x_k^T\right)C_k^T\end{pmatrix}\mid Y_t\right)\end{aligned} \quad (11)$$

where $B_t^2 = B_t B_t^T$, $D_t^2 = D_t D_t^T$, and $E(\cdot)$ denotes the expectation operator. The system parameters $\left\{\hat{A}_t, \hat{B}_t^2, \hat{C}_t, \hat{D}_t^2\right\}$ can be computed from the conditional expectations as [13]:

$$\begin{aligned}L_t^{(1)} &= E\left\{\sum_{k=1}^{t} x_k^T Q x_k \mid Y_t\right\},\ L_t^{(2)} = E\left\{\sum_{k=1}^{t} x_{k-1}^T Q x_{k-1} \mid Y_t\right\}\\ L_t^{(3)} &= E\left\{\sum_{k=1}^{t}\left[x_k^T R x_{k-1} + x_{k-1}^T R^T x_k\right]\mid Y_t\right\}\\ L_t^{(4)} &= E\left\{\sum_{k=1}^{t}\left[x_k^T S y_k + y_k^T S^T x_k\right]\mid Y_t\right\}\end{aligned} \quad (12)$$

where Q, R and S are given by:

$$\begin{aligned}Q &= \left\{\frac{e_i e_j^T + e_j e_i^T}{2};\ i,j = 1,2,...n\right\}, R = \left\{\frac{e_i e_j^T}{2};\ i,j = 1,2,...n\right\}\\ S &= \left\{\frac{e_i e_j^T}{2};\ i = 1,2,...n;\ j = 1,2,..d\right\}\end{aligned} \quad (13)$$

in which e_i is the unit vector in the Euclidean space; that is $e_i = 1$ in the ith position, and 0 elsewhere. For instance, consider the case $n = d = 2$, then $E\left(\sum_{k=1}^{t} x_k x_{k-1}^T \mid Y_t\right)$ is:

$$E\left(\sum_{k=1}^{t} x_k x_{k-1}^T \mid Y_t\right) = \begin{bmatrix} L_t^{(3)}(R_{11}) & L_t^{(3)}(R_{12})\\ L_t^{(3)}(R_{21}) & L_t^{(3)}(R_{22})\end{bmatrix} \quad (14)$$

where $R_{ij} = \left\{e_i e_j^T/2;\ i,j = 1,2\right\}$. The other terms in (11) can be computed similarly.

The conditional expectations $\left\{L_t^{(1)}, L_t^{(2)}, L_t^{(3)}, L_t^{(4)}\right\}$ can be estimated from measurements Y_t as follows:

1) Filter estimate of $L_t^{(1)}$:

$$\begin{aligned}L_t^{(1)} &= E\left\{\sum_{k=1}^{t} x_k^T Q x_k \mid Y_t\right\}\\ &= -\frac{1}{2}Tr\left(N_t^{(1)}P_{t|t}\right) - \frac{1}{2}\sum_{k=1}^{t} Tr\left(N_{k-1}^{(1)}\overline{P}_{k|k}\right)\\ &-\frac{1}{2}\sum_{k=1}^{t}\begin{pmatrix}-2x_{k|k}^T P_{k|k}^{-1} r_k^{(1)} + 2x_{k|k-1}^T P_{k|k-1}^{-1} r_{k|k-1}^{(1)} - x_{k|k}^T N_k^{(1)} x_{k|k}\\ +x_{k|k-1}^T B_k^{-2} A_k \overline{P}_{k|k} N_{k-1}^{(1)} \overline{P}_{k|k} A_k^T B_k^{-2} x_{k|k-1}\end{pmatrix}\end{aligned} \quad (15)$$

where $Tr(\cdot)$ denotes the matrix trace. In (15), $r_k^{(1)}$ and $N_k^{(1)}$ satisfy the following recursions:

$$\begin{aligned}&\begin{cases} r_k^{(1)} = \left(A_k - P_{k|k}C_k^T D_k^{-2} C_k A_k\right) r_{k-1}^{(1)} + 2P_{k|k} Q x_{k|k-1}\\ \qquad - P_{k|k} N_k^{(1)} P_{k|k} C_k^T D_k^{-2}\left(y_k - C_k x_{k|k-1}\right)\\ r_{k|k-1}^{(1)} = A_k r_k^{(1)}\\ r_0^{(1)} = 0_{m\times 1}\end{cases}\\ &\begin{cases} N_k^{(1)} = B_k^{-2} A_k \overline{P}_{k|k} N_{k-1}^{(1)} \overline{P}_{k|k} A_k^T B_k^{-2} - 2Q\\ N_0^{(1)} = 0_{m\times m}\end{cases}\end{aligned} \quad (16)$$

2) Filter estimate of $L_t^{(2)}$:

$$\begin{aligned}L_t^{(2)} &= E\left\{\sum_{k=1}^{t} x_{k-1}^T Q x_{k-1} \mid Y_t\right\}\\ &= E_\theta\left\{x_0^T Q x_0 \mid Y_t\right\} + E_\theta\left\{\sum_{k=1}^{t} x_k^T Q x_k \mid Y_t\right\} - E_\theta\left\{x_t^T Q x_t \mid Y_t\right\}\end{aligned} \quad (17)$$

Therefore, $L_t^{(2)}$ can be obtained from $L_t^{(1)}$.

3) Filter estimate of $L_t^{(3)}$:

$$\begin{aligned}L_t^{(3)} &= E\left\{\sum_{k=1}^{t}\left(x_k^T R x_{k-1} + x_{k-1}^T R^T x_k\right)\mid Y_t\right\}\\ &= -\frac{1}{2}Tr\left(N_t^{(3)}P_{t|t}\right) - \frac{1}{2}\sum_{k=1}^{t} Tr\left(N_{k-1}^{(3)}\overline{P}_{k|k}\right)\\ &-\frac{1}{2}\sum_{k=1}^{t}\begin{pmatrix}-2x_{k|k}^T P_{k|k}^{-1} r_k^{(3)} + 2x_{k|k-1}^T P_{k|k-1}^{-1} r_{k|k-1}^{(3)} - x_{k|k}^T N_k^{(3)} x_{k|k}\\ +x_{k|k-1}^T B_k^{-2} A_k \overline{P}_{k|k} N_{k-1}^{(3)} \overline{P}_{k|k} A_k^T B_k^{-2} x_{k|k-1}\end{pmatrix}\end{aligned} \quad (18)$$

In this case, $r_k^{(3)}$ and $N_k^{(3)}$ satisfy the following recursions:

$$\begin{cases} r_k^{(3)} = \left(A_k - P_{k|k} C_k^T D_k^{-2} C_k A_k\right) r_{k-1}^{(3)} - P_{k|k} N_k^{(3)} P_{k|k} C_k^T D_k^{-2} \\ \qquad \left(y_k - C_k x_{k|k-1}\right) + \left(2P_{k|k} R + 2P_{k|k} B_k^{-2} A_k \overline{P}_{k|k} R^T A_k\right) x_{k-1|k-1} \\ r_{k|k-1}^{(3)} = A_k r_k^{(3)} \\ r_0^{(3)} = 0_{m\times 1} \end{cases} \tag{19}$$

$$\begin{cases} N_k^{(3)} = B_k^{-2} A_k \overline{P}_{k|k} N_{k-1}^{(3)} \overline{P}_{k|k} A_k^T B_k^{-2} - 2R\overline{P}_{k|k} A_k^T B_k^{-2} \\ \quad -2B_k^{-2} A_k \overline{P}_{k|k} R^T \\ N_0^{(3)} = 0_{m\times m} \end{cases}$$

4) Filter estimate of $L_t^{(4)}$:

$$\begin{aligned} L_t^{(4)} &= E\left\{\sum_{k=1}^{t}\left(x_k^T S y_k + y_k^T S^T x_k\right) \mid Y_t\right\} \\ &= \sum_{k=1}^{t}\left(x_{k|k}^T P_{k|k}^{-1} r_k^{(4)} - x_{k|k-1}^T P_{k|k-1}^{-1} r_{k|k-1}^{(4)}\right) \end{aligned} \tag{20}$$

where $r_k^{(4)}$ satisfy the following recursions:

$$\begin{cases} r_k^{(4)} = (A_k - P_{k|k} C_k^T D_k^{-2} C_k A_k) r_{k-1}^{(4)} + 2P_{k|k} S y_k \\ r_{k|k-1}^{(4)} = A_k r_k^{(4)} \\ r_0^{(4)} = 0_{m\times 1} \end{cases} \tag{21}$$

Using the filters for $L_t^{(i)}$ $(i = 1,2,3,4)$ and the Kalman filter described earlier, the system parameters $\theta_t = \{A_t, B_t, C_t, D_t\}$ can be estimated through the EM algorithm described in (11). Numerical results that show the applicability of the above algorithm in estimating the channel parameters as well as the inphase and quadrature components from measurements are discussed in Section V. In the next section, we introduce one important application based on the developed models, which is stochastic PC in wireless networks.

IV. Stochastic Power Control Algorithm Based on the Developed Wireless Channel Models

In this section, a PCA based on the estimated wireless fading channel models is developed. Since the channel model parameters are estimated from received signal measurements, PC can be performed just from having these measurements. The aim of the PCA described here is to minimize the total transmitted power of all users while maintaining acceptable QoS for each user. The measure of QoS is defined by the SIR for each link to be larger than a target SIR.

Now consider a wireless network with M transmitters and N receivers. The state space representation of a flat fading wireless network can be written as:

$$\begin{aligned} dX_{ij}(t) &= A_{ij} X_{ij}(t)dt + B_{ij} dW_{ij}(t) \\ y_i(t) &= \sum_{k=1}^{M} \sqrt{p_k(t)} s_k(t) C_{ik}(t) X_{ik}(t) + n_i(t) \end{aligned} \tag{22}$$

where $y_i(t)$ is the received signal at the ith receiver at time t, $X_{ik}(t)$ is the states of the channel between transmitter k and the receiver assigned to transmitter i, $p_k(t)$ is the transmitted power of transmitter k at time t, which acts as a scaling on the information signal $s_k(t)$, $n_i(t)$ is the channel disturbance or noise at receiver i, and $1 \le i, j \le M$.

Consider the wireless network described above, the centralized PC problem for TV channels over a time interval $[0,T]$ can be stated as follows [1]:

$$\min_{(p_1 \ge 0,\ldots,p_M \ge 0)} \left\{\sum_{i=1}^{M} \int_0^T p_i(t)\,dt\right\}, \quad \text{subject to}$$

$$\frac{\int_0^T p_i(t) s_i^2(t) \left[C_{ii}(t) X_{ii}(t)\right]^2 dt}{\sum_{k\ne i}^{M} \int_0^T p_k(t) s_k^2(t)\left[C_{ik}(t) X_{ik}(t)\right]^2 dt + \int_0^T n_i^2(t)\,dt} \ge \varepsilon_i \tag{23}$$

and $i = 1, \cdots, M$. A solution to (23) is presented by first introducing the communication meaning of predictable power control strategies (PPCS). In wireless cellular networks, it is practical to observe and estimate channels at base stations and then send the information back to the mobiles to adjust their power signals $\{p_i(t_k)\}_{i=1}^M$. Since channels experience delays, and power control is not feasible continuously in time but only at discrete time instants, the concept of predictable strategies is introduced [1]. Consider a set of discrete time strategies $\{p_i(t_k)\}_{i=1}^M$, $0 = t_0 < t_1 < \ldots < t_k < t_{k+1} < \ldots \le T$. At time t_{k-1}, the base stations estimate the channel information $\{I_{ij}(t_{k-1}), Q_{ij}(t_{k-1}), s_i(t_{k-1})\}_{i,j=1}^M$ as described in Section III. Using the concept of predictable strategy, the base stations determine the control strategy $\{p_i(t_k)\}_{i=1}^M$ for the next time instant t_k. The latter is communicated back to the mobiles, which hold these values during the time interval $[t_{k-1}, t_k)$. At time t_k, a new set of channel information $\{I_{ij}(t_{k-1}), Q_{ij}(t_{k-1}), s_i(t_{k-1})\}_{i,j=1}^M$ is estimated at the base stations and the time t_{k+1} control strategies $\{p_i(t_{k+1})\}_{i=1}^M$ are computed and communicated back to the mobiles which hold them constant during the time interval $[t_k, t_{k+1})$. Such decision strategies are called predictable. Using the concept of PPCS over any time interval $[t_k, t_{k+1}]$, equation (23) is equivalent to:

$$\begin{aligned} &\min_{\mathbf{p}(t_{k+1})>0} \sum_{i=1}^{M} p_i(t_{k+1}) \quad \text{subject to} \\ &\mathbf{p}(t_{k+1}) \ge \mathbf{\Gamma G}_I^{-1}(t_k, t_{k+1}) \times \left(\mathbf{G}(t_k, t_{k+1})\mathbf{p}(t_{k+1}) + \boldsymbol{\eta}(t_{k+1})\right) \end{aligned} \tag{24}$$

where

$$g_{ij}(t_k, t_{k+1}) := \int_{t_k}^{t_{k+1}} s_j^2(t)\left[C_{ij}(t) X_{ij}(t)\right]^2 dt, \; 1 \le i, j \le M,$$

$$\mathbf{p}(t_{k+1}) := \left(p_1(t_{k+1}), \cdots, p_M(t_{k+1})\right)^T, \; \eta_i(t_k, t_{k+1}) := \int_{t_k}^{t_{k+1}} n_i^2(t)\,dt,$$

$$\mathbf{G}_I(t_k,t_{k+1}) := diag\left(g_{11}(t_k,t_{k+1}),\cdots,g_{MM}(t_k,t_{k+1})\right),$$

$$\mathbf{G}(t_k,t_{k+1}) := \begin{cases} 0 & ,\text{ if } i=j \\ g_{ij}(t_k,t_{k+1}) & ,\text{ if } i\neq j \end{cases}, 1\le i,\ j\le M,$$

$$\boldsymbol{\eta}(t_k,t_{k+1}) := \left(\eta_1(t_k,t_{k+1}),\cdots,\eta_M(t_k,t_{k+1})\right)^T,\ \boldsymbol{\Gamma} := diag(\varepsilon_1,\cdots,\varepsilon_M),$$

and $diag(\cdot)$ denotes a diagonal matrix with its argument as diagonal entries. The optimization in (24) is a linear programming problem in $M\times 1$ vector of unknowns $\mathbf{p}(t_{k+1})$. Here $[t_k, t_{k+1}]$ is a time interval such that the channel model does not change significantly, i.e., $[t_k, t_{k+1}]$ should be smaller than the coherence time of the channel. Throughout this section, we assume that the PC problem is feasible, i.e., there exists a power vector $\mathbf{p}(t_k)$ that satisfies the inequality in (24) for all $[t_k, t_{k+1}]$ in $[0,T]$. Note that the PCAs in (24) can be used as long as the channel model does not change significantly, that is $[t_k, t_{k+1}]$ is a subset of the coherence time of the channel.

In the next section, a numerical example is presented to determine the performance of the proposed PCA under the estimated wireless channel models.

V. Numerical Examples

In this section, two numerical examples are presented. In Example 1, the developed EM algorithm together with Kalman filtering is performed to estimate channel parameters as well as inphase and quadrature components from the received signal measurements. In Example 2, the performance of the proposed PCA based on the estimated channel models is determined and compared with the one of fixed transmitted powers.

Example 1:

In this example, a 4th order channel model as described in (6) and (7) is considered. Therefore, the system parameters $\theta_t = \{A_t, B_t, C_t, D_t\}$ can be represented as:

$$A_t = \begin{bmatrix} 0 & 1 & 0 & 0 \\ a_1 & a_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & a_3 & a_4 \end{bmatrix}, B_t = \begin{bmatrix} b_1 & \delta_{12} & \delta_{13} & \delta_{14} \\ b_2 & \delta_{22} & \delta_{23} & \delta_{24} \\ \delta_{31} & \delta_{32} & b_3 & \delta_{34} \\ \delta_{41} & \delta_{42} & b_{43} & \delta_{44} \end{bmatrix}, \quad (25)$$

$$C_t = \begin{bmatrix} \cos(\omega_c t) & 0 & -\sin(\omega_c t) & 0 \end{bmatrix}, D_t = \begin{bmatrix} d_1 & d_2 \end{bmatrix}$$

As previously mentioned, the estimation of a flat fading wireless channel from received signal measurement data is considered. In particular, the estimation includes the channel parameters, inphase and quadrature components, and the received signal, which are then compared to the ones obtained from measurement data.

It is assumed that the received signal measurement data are corrupted by white noise sequences. Figure (1) shows the measured and estimated inphase and quadrature components as well as the received signal using the EM algorithm together with Kalman filter for 400 sampled data taken from the measurements of one channel chosen at random. From Figure (1), it can be noticed that the inphase and quadrature components of the wireless fading channel as well as the received signal have been estimated with very high accuracy. It can also be noticed that the estimation error decreases as the number of samples increases; this is because the algorithm is recursive and the channel parameters converge to the actual values as more samples are being estimated.

Example 2:

In this example, the received signal measurement data for 24 users are collected experimentally. They represent flat Rayleigh fading environment where the signal envelope at the receivers exhibit Rayleigh distributed density. The channel model parameters as well as the inphase and quadrature components for all users are estimated online from the measurement data using the EM algorithm together with the Kalman filter as illustrated in Example 1. The PCA described in (24) is performed using the estimated channel parameters and states. The outage probability (OP) is used as a performance measure for the PCA. A link with a received SIR R_i, less than or equal to a target SIR ε_i, is considered a communication failure. The OP, $O(\varepsilon_i)$, is expressed as $O(\varepsilon_i) = \text{Prob}\{R_i \le \varepsilon_i\}$, where R_i is the received SIR at receiver *i*.

It is assumed that the targets SIR ε_i for all users are the same, and varied from 5 dB to 25 dB with step 5 dB. For each value of ε_i the OP is computed every 15 millisecond, i.e., $[t_k, t_{k+1}] = 15$ millisecond. The simulation is performed for 4.5 seconds, i.e., $[0, T] = 4.5$ seconds. The OP is computed using Monte-Carlo simulations. The performance of the proposed PCA is compared with the one of constant transmitted powers (CTP).

The OP for both the CTP and the proposed PCA based on PPCS are shown in Figure (2a) and (2b), respectively. Figure 2 shows how the OP changes with respect to the target SIR, ε_i, and time. As the target SIR increases the OP increases. This is obvious since we expect more users to fail.

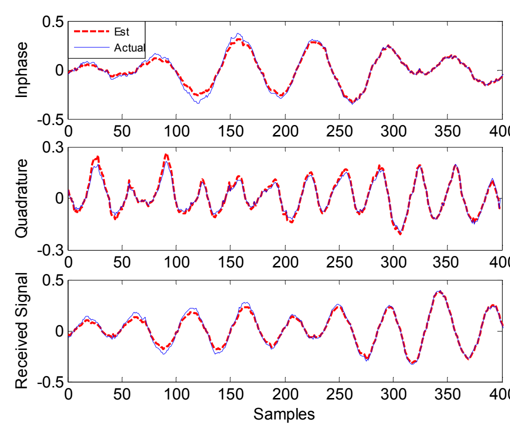
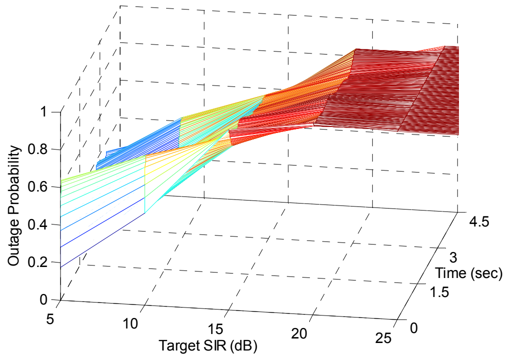
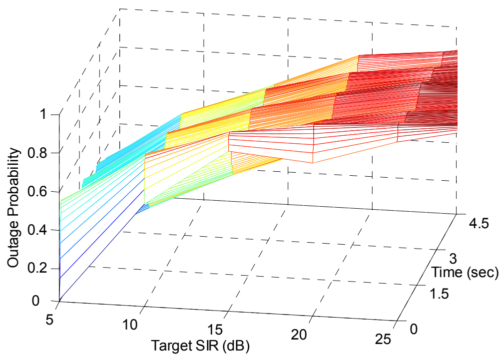


Fig. 1. The real and estimated inphase and quadrature components, and received signal for 4th order channel model in Example 1, using the EM algorithm together with Kalman filter.



(a)



(b)

Fig. 2. OP for dynamical flat Rayleigh wireless network in Example 2. (a) Using CTP. (b) Using PC based on PPCS.

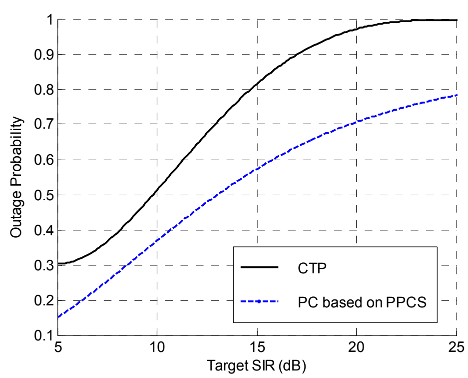


Fig. 3. Average OP for the PC case which is based on PPCS and the CTP case. Performance comparison.

The OP also changes as a function of time, since users move in different directions and velocities while taking the measurements.

The average OP versus ε_i for both cases over the whole simulation time (4.5 seconds) is shown in Figure (3), which shows that the performance of the PCA based on PPCS is on average much better than that of the CTP. For example, at 10 dB target SIR, the OP is reduced from 0.51 for the CTP algorithm to 0.37 for the PCA; this represents an improvement of over 27%. The PCA based on PPCS outperforms the CTP case by an order of magnitude.

VI. Conclusion

This paper describes a general scheme for extracting mathematical channel models from noisy received signal measurements, and performing power control based on the estimated channel parameters. The channel models are represented in state space form. The proposed algorithm consists of filtering based on the Kalman filter to remove noise from data, and identification based on the Expectation Maximization (EM) algorithm to determine the parameters of the model which best describe the measurements. Numerical results indicate that the measured data can be generated through a simple 4th order discrete time stochastic differential equation. Moreover, a stochastic power control algorithm (PCA) based on the estimated parameters and channel states is proposed. Numerical results indicate that there is potentially large gain to be achieved by using PC. The proposed PCA can be used as long as the channel model does not change significantly, that is $[t_k, t_{k+1}]$ is a subset of the coherence time of the channel.

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