Efficient Implementation of the Chan-Vese Models Without Solving PDEs

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***Abstract*— Efficient implementation methods are proposed for Chan-Vese models [3] [16]. The proposed methods do not require solutions of PDEs and are therefore fast. The advantages of level set methods, such as automatic handling of topological changes, are preserved. These methods utilize region information to guide the evolution of initial curves. Gaussian smoothing is applied to regularize the evolving curves. These algorithms are able to automatically and efficiently segment objects in complicated images. Experimental results show that the proposed methods work efficiently for images without strong noise. However, they still have initialization problems, as do the Chan-Vese models.**

I. Introduction

Curve evolution methods [1] [2] [3] [4] [6] [7] [9] [10] [13] [14] [15] [16] [17] are widely used in image segmentation problems. These methods drive one or more initial curve(s), based on gradient and/or region information in the image, to the boundaries of objects in that image. These methods are derived using variational methods, and are implemented using finite difference approximations to PDEs and level sets [5] [12].

In curve evolution methods, region-based geometric methods [3] [4][9] [13] [15] [16] have several advantages. First, they can deal with topological changes automatically, outperforming parametric methods such as [6] and [17]. Second, utilization of the global region information stabilizes their responses to local variations (such as weak edges and noise) in comparison to gradient-based geometric methods [1] [2] [7] [10] [14].

Region-based geometric methods, however, have some limitations. First, these methods are usually implemented by solving PDEs, and are thus computationally intense. Second, most have initialization problems [4]: different initial curves produce different segmentations.

Efficient implementation methods for the Chan-Vese models are proposed in this paper. The methods do not have to solve PDEs. The computational load of curve evolution is thus greatly reduced. The proposed methods bear some similarities to [14], but they are region-based rather than gradient-based. It is more straightforward to build region information to drive curve evolution. More complicated issues that are not considered in [14], such as sensitivity to noise, are discussed. The proposed methods are still sensitive to the selection of initial curves, as are the Chan-Vese models, but they work efficiently for images without strong noise. They can also deal with complicated images such as triple junctions.

The paper is organized as follows. In section II, the Chan-Vese models are introduced and their implementations are discussed. Fast curve evolution methods that do not require solutions of PDEs are proposed in section III. Experimental results are given and analyzed in section IV. Section V provides a summary with conclusions and future work.

II. Introduction to the Chan-Vese Models

The bi-modal Chan-Vese model is reviewed first, followed by the multi-phase Chan-Vese model.

A. The Bi-modal Chan-Vese Model

The Chan-Vese models [3] [16] are curve evolution implementations of a well-posed case of the Mumford-Shah model [11]. The bi-modal Chan-Vese model [3] segments an image by solving the PDE

$$\psi_t = \delta_\epsilon(\psi)[\mu \cdot \kappa - (I - c_1)^2 + (I - c_2)^2] \qquad (1)$$

where I is the original image, ψ is the level set representation of the evolving curve C, which means $C = \{(x, y)|\psi(x, y) = 0\}$. c_1 and c_2 are selected as the average values of pixels inside and outside C, respectively. κ represents the curvature of the evolving curve. $\delta_\epsilon(\psi) = \epsilon/(\pi(\epsilon^2 + \psi^2))$ and ϵ is a positive constant.

From (1), the evolution of the curve is influenced by two terms. The curvature term κ regularizes the curve and makes it smooth during evolution. The region term $-(I-c_1)^2+(I-c_2)^2$ affects the motion of the curve. The initialization of the curve affects curve evolution through this term.

B. The Multiphase Chan-Vese Model

The bi-modal Chan-Vese model is applicable only for bi-modal images. The multiphase Chan-Vese model [16] has been proposed for more complex images. In this model, two or more coupled curves evolve simultaneously to segment images with multiple objects. Consider a four-phase Chan-Vese model, in which two coupled curves ψ_1 and ψ_2 evolve according to coupled Euler-Lagrange equations.

Suppose the initial curves divide the image into four regions: $R_{00} = \{\psi_1 < 0, \psi_2 < 0\}$, $R_{10} = \{\psi_1 > 0, \psi_2 < 0\}$, $R_{01} = \{\psi_1 < 0, \psi_2 > 0\}$, $R_{11} = \{\psi_1 > 0, \psi_2 > 0\}$, as shown in Fig. 1 (A). Let c_{00}, c_{10}, c_{01}, and c_{11} be the average intensities

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inside R_{00}, R_{10}, R_{01}, R_{11}, respectively. The evolution of ψ_1 and ψ_2 follows the Euler-Lagrange equations:

$$\frac{\partial \psi_1}{\partial t} = \delta_\epsilon(\psi_1)\{\nu\kappa_1 - ((I_0 - c_{11})^2 - (I_0 - c_{01})^2)H(\psi_2) - ((I_0 - c_{10})^2 - (I_0 - c_{00})^2)(1 - H(\psi_2))\} \tag{2}$$

$$\frac{\partial \psi_2}{\partial t} = \delta_\epsilon(\psi_2)\{\nu\kappa_2 - ((I_0 - c_{11})^2 - (I_0 - c_{10})^2)H(\psi_1) - ((I_0 - c_{01})^2 - (I_0 - c_{00})^2)(1 - H(\psi_1))\} \tag{3}$$

where $\kappa_1 = \nabla\cdot(\frac{\nabla\psi_1}{|\nabla\psi_1|})$ and $\kappa_2 = \nabla\cdot(\frac{\nabla\psi_2}{|\nabla\psi_2|})$ are the curvatures of the evolving curve ψ_1 and ψ_2. $H(\cdot)$ is the Heaviside function: $H(x) = 1$ when $x > 0$ and $H(x) = 0$ when $x < 0$.

It can be seen from (2) that the evolution of ψ_1 determines a boundary comprised of two parts: the part between R_{00} and R_{10} where $\psi_2 < 0$, and the part between R_{01} and R_{11} where $\psi_2 > 0$. The first part evolves due to region information in R_{00} and R_{10}. The evolution of the second part is driven by region information between R_{01} and R_{11}. Similar observations can be made for ψ_2. In this manner, the multiphase Chan-Vese model divides the image into several smaller regions and performs curve evolution based on region information in these regions.

III. Fast Curve Evolution Methods Without Solving PDEs

The Chan-Vese models are usually implemented by solving PDEs, such as the level set equations [5] [12] and Poisson equations [16]. These methods are computationally intense, although they are theoretically sound. Fast implementation methods are proposed for both the bi-modal and the multiphase Chan-Vese models in this section. A mathematical analysis of the region information in the bimodal Chan-Vese model is provided first, which provides the concept for a fast implementation evolution method that does not require solving PDEs. The proposed method is extended to the decoupled multiphase Chan-Vese model, as provided in [4].

A. *Region Information in the Bi-modal Chan-Vese Model*

Consider a piecewise constant bi-modal image. Suppose there are n_1 pixels in the background of the image, among which $m_1 (0 \le m_1 \le n_1)$ pixels lie inside the initial curve. Suppose there are n_2 pixels in the foreground of the image, among which $m_2 (0 \le m_2 \le n_2)$ lie inside the initial curve. All the pixels in the background (foreground) take $u_1(u_2)$ as their intensity values. Obviously, $m_1 + m_2 > 0$ for all initializations.

Therefore, the region terms $R = -(I - c_1)^2 + (I - c_2)^2$ for points on the evolving curve in the foreground and the background are,

$$(u_2 - c_2)^2 - (u_2 - c_1)^2 = K_0 K_2 (m_2 n_1 - m_1 n_2)(u_1 - u_2)^2 \tag{4}$$

$$(u_1 - c_2)^2 - (u_1 - c_1)^2 = -K_0 K_1 (m_2 n_1 - m_1 n_2)(u_1 - u_2)^2 \tag{5}$$

where $K_0 = 1/\{(n_1 - m_1 + n_2 - m_2)(m_1 + m_2)\}$, $K_1 = (n_2 - m_2)/(n_1 - m_1 + n_2 - m_2) + m_2/(m_1 + m_2)$ and $K_2 = (n_1 - m_1)/(n_1 - m_1 + n_2 - m_2) + m_1/(m_1 + m_2)$, respectively. K_0, K_1 and K_2 are positive for any initialization.

From the equations (4) and (5), the following observations can be acquired for the bi-modal Chan-Vese model: First, the region term $R = -(I - c_1)^2 + (I - c_2)^2$ of the foreground has the opposite sign to the region term of the background for any initialization. Second, for any point strictly inside the foreground or the background, its region term will has the same sign as the terms for its neighboring points. Third, only the boundary points will have neighboring points with region terms different in sign.

B. *Fast Implementation of the Bi-modal Chan-Vese Model*

The observations shown in the last subsection hold for all bi-modal images without strong noise. Furthermore, they hold independent of the initial position of the evolving curve. This information can thus be used to construct an efficient implementation of the bi-modal Chan-Vese model. In the implementation, a list of points on the curve C, instead of a narrow band of points around the curve in classical methods [5] [12], is utilized to represent the evolving curve. The key idea is to evolve the curve C until it reaches a position where neighboring points have region terms R different in sign. The evolving curve is updated by removing the points whose neighboring points have region terms with the same sign (since these points are not boundary points) and inserting neighboring points, based on the direction of curve evolution, which is determined by the signs of ψ and R.

Without loss of generality, suppose the points in the region inside the initial curve are set to have positive ψ values. For any point in the list, if this point and all of its neighboring points have positive region terms, i.e. $R > 0$, then from (1) the ψ value of this point will increase and the curve will expand at this point. We only need to remove this point from the list, and add to the list those neighboring points lying in the region outside the evolving curve C, i.e. those neighboring points that have negative ψ values. Correspondingly, if a point in the list and all its neighboring points have negative region terms, i.e. $R < 0$, then the curve will shrink at that point from (1). We need to remove this point from the list and add to the list those neighboring points lying in the region inside the evolving curve, i.e. those neighboring points that have positive ψ values. Otherwise, this point and its neighboring points have region terms different in sign, and the boundary has been reached. The evolving curve will stop at this point. The computational load of the proposed method is low since only a list of points on the curve is updated based on region terms R. No PDEs need to be solved.

Up to this point, the regularization term in (1) has not been used in the proposed method. As proved in [8], curve evolution based on curvature $\partial C/\partial t = \kappa N$ is equivalent to a nonlinear analogy to Gaussian smoothing. Thus, Gaussian smoothing can be applied after each iteration of curve evolution to smooth the curve.

C. Fast Implementation of the Multiphase Chan-Vese Model

It seems straightforward to extend the proposed method to the multiphase Chan-Vese model. In this case, two or more curves are initialized and then evolved according to the region information. Gaussian smoothing is utilized to regularize the evolving curves. However, the coupling between the evolving curves in the multiphase Chan-Vese model, as can be seen from (2) and (3), may cause the evolving curves to stop at a local minimum [4] [16].

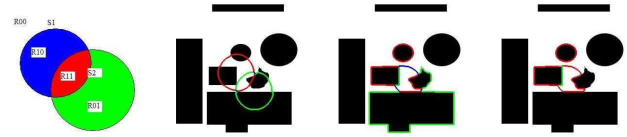
To illustrate this effect, Fig. 1(B) shows an image with initialized curves. Let ψ_1 be the red curve and ψ_2 be the green curve. Both ψ_1 and ψ_2 are initialized to be positive inside the curve and negative outside the curve. Fig. 1(C) shows the segmentation result using the coupled multiphase Chan-Vese model. The blue lines in Fig. 1(C) means the red curve and the green curve both stop there. It can be seen that segments of the evolving curves do not correspond to object boundaries, and only a local minimum is reached. (D) shows the final location of the green segment of the evolving curve, which corresponds to the portion of ψ_2 in the region $\psi_1 > 0$. This part of the curve, which evolves under the direction of region information in the area of $\psi_1 > 0$, reaches its local minimum. Similar results can be achieved for the other three parts of the evolving curves.

This problem arises from the coupling of the evolving curves. Decoupled models [4] have been proposed to solve the problem. In this model, only one curve is evolved at a time. The first curve ψ_1 separates the original image into two regions $\{\psi_1 > 0\}$ and $\{\psi_1 < 0\}$. The second curve ψ_2 evolves based on the results of the first curve, and may segment the original image into three or four regions, such as $\{\psi_1 > 0, \psi_2 < 0\}$. This procedure is repeated until all the objects in the original image are segmented. The evolving curves in decoupled models are more likely to stop at object boundaries because they do not evolve simultaneously.

Decoupled models in [4] reduce the effects of coupling, but they solve PDEs for image segmentation, which is computationally intense. The fast implementation method proposed in the above section can be extended for decoupled multiphase models. Consider the four-phase case of the Chan-Vese model. Two curves C_1 and C_2 are initialized, and evolve in consecutive iterations. In each iteration, the implementation method for the bi-modal case is utilized. For the first iteration, the curve C_1 evolves using region information $R_1 = ((I_0 - c_{11})^2 - (I_0 - c_{01})^2)H(\psi_2) - ((I_0 - c_{10})^2 - (I_0 - c_{00})^2)(1 - H(\psi_2))$ as in Eqn. (2). After the first iteration is completed, the other curve C_2 evolves using region information $R_2 = -((I_0 - c_{11})^2 - (I_0 - c_{10})^2)H(\psi_1) - ((I_0 - c_{01})^2 - (I_0 - c_{00})^2)(1 - H(\psi_1))$ as in Eqn. (3). Gaussian smoothing is utilized for regularization.

IV. Experimental Results

Experimental results from the proposed method are shown in this section. The proposed method is implemented on a computer which has two Intel(R) Pentium(R) 3.2GHz CPUs, 2G bytes RAM, and runs the Red Hat Enterprise Linux operating system. The CPU times given in this paper are the



(A) (B) (C) (D)

Fig. 1

Coupling between curve evolution may enlarge initialization problems. (A) Multiphase Chan-Vese model. (B) Initialization. (C) Segmentation results. (D) show the positions of one part of the evolving curves in (C).

sums of system CPU times and user CPU times. The system CPU time is usually very small, typically 0.01 - 0.08 seconds.

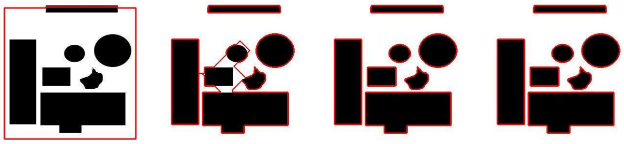
The proposed method is efficient compared to the classical method solving PDEs [5] [12], as can be seen from Fig. 2, which shows that the proposed method is 23 times faster and achieves the same segmentation results. Fig. 2 (A)-(C) demonstrate that the proposed method is able to automatically handle topological changes.

The performance of the proposed method for the decoupled multiphase model is illustrated in Fig. 3. For the initialization in (A), the red curve as shown in (B) is evolved first, and the result is shown in (C). The green curve shown in (D) is evolved afterward, and its result is provided in (E). The final segmentation of the original image is shown in (F). (G) and (H) show another initialization and the segmentation result separately. It can be seen that the initialization problem still exists.

Fig. 4 shows segmentation results for complicated images. Fig. 4 (A1) and (A2) show that the proposed bi-modal method works for images with weak edges. The results in Figs. 4 (B1)(B2) demonstrate the ability of the method to handle local variations. Fig. 4 (C1) - (C4) shows the segmentation of triple junctions using the proposed method for the decoupled multiphase Chan-Vese model.

The effects of noise on the proposed method are illustrated in Fig. 5. Fig. 5 (A)-(C) show the segmentation of an image with medium noise. Although the noise affects the process of curve evolution, as can be seen in Fig. 5 (B), the object in the image is successfully segmented in Fig. 5 (C). In this case region information has to be updated after every iteration to reduce the effects of noise, which is not required in the image of Fig. 2. Fig. 5 (D) shows the curve evolution result after 100 iterations for an image with strong noise. The proposed method fails in this case. The reason is that the noise in the image is so strong that it changes the sign of the region term R. The curve can not evolve in the proper direction from the sign of R.

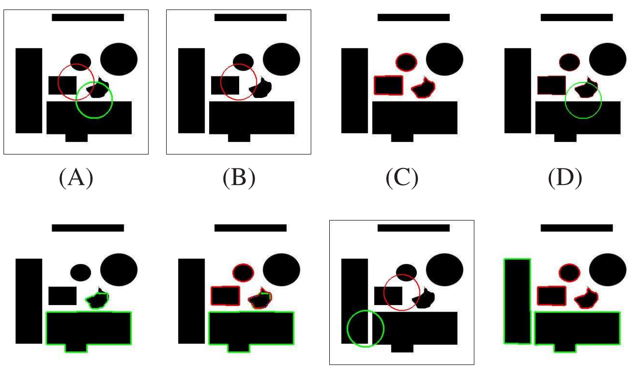
Fig. 6 demonstrates the performance of the proposed method relative to the classical implementation of the bi-modal Chan-Vese model on real images. (A1-D1) shows the images to be segmented. (A2-D2) represents the initialized curves. (A3-D3) are the segmented images using the proposed methods for the bi-modal Chan-Vese models. (A4-D4) are the segmentations using the classical bi-modal Chan-Vese models. It can be seen that the segmentation results from the methods are very similar, but the proposed method is much faster.



(A) (B) (C) (D)

Fig. 2

COMPARISON OF THE PROPOSED FAST CURVE EVOLUTION METHOD TO THE CLASSICAL METHOD SOLVING PDES [5] [12]. (A) AN IMAGE (300 * 300) WITH INITIAL CURVE. (B) INTERMEDIATE RESULT USING THE PROPOSED METHOD. (C) SEGMENTATION RESULT OF THE PROPOSED METHOD, CPU = 0.51S. (B)(D) SEGMENTATION RESULT OF THE CHAN-VESE MODEL BY SOLVING PDES, CPU = 11.98S. THE NEW METHOD PROVIDES A 23 TIMES SPEED-UP.



(A) (B) (C) (D)

(E) (F) (G) (H)

Fig. 3

FAST IMPLEMENTATION OF THE DECOUPLED MULTIPHASE CHAN-VESE MODEL. (A) AN IMAGE (300 * 300) WITH INITIAL CURVE. (B)(C) THE FIRST ITERATION AND THE RESULT. (D)(E) THE SECOND ITERATION AND THE RESULT. (F) FINAL SEGMENTATION. CPU = 0.514S. (G)(H) ANOTHER INITIALIZATION AND ITS RESULT. CPU = 5.416S.

V. CONCLUSIONS AND FUTURE WORK

In this paper, fast implementation methods for the Chan-Vese models are proposed that do not require solution of the PDEs. These methods utilize region information to guide the evolution of the evolving curves and Gaussian smoothing to regularize them. Experimental results show that these methods work very efficiently for images with no strong noise. If noise is strong enough to change the signs of the region terms, the proposed algorithm may fail.

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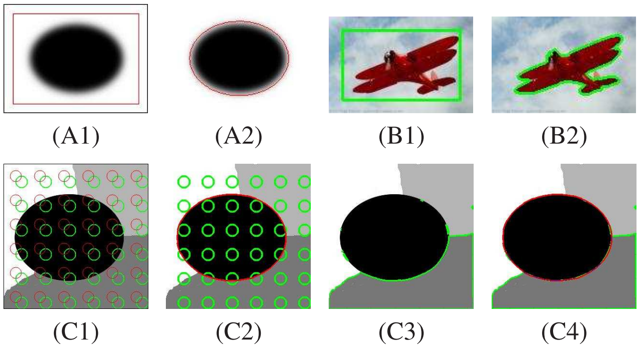

Fig. 4

Fast curve evolution for complicated cases. (A1) An image with weak edges and the initial curve (200 * 150). (A2) Segmentation result of (A1) using the bi-modal Chan-Vese model, CPU = 0.28s. (B1) A real image with the initial curve (200 * 133). (B2) Segmentation result of (B1) using the bi-modal Chan-Vese model, CPU = 0.08s.(C1-C4) Segmentation of triple junctions (300 * 300) using the decoupled multiphase Chan-Vese model. CPU = 1.960s.

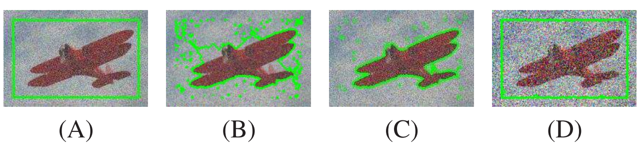

Fig. 5

Effects of noise on the proposed method. (A) An image with medium noise. (B) Curve evolution after 44 iterations. (C) Segmentation results of (A), CPU = 0.3s. (D) Curve evolution after 100 iterations for an image with strong noise. The proposed method fails in (D).

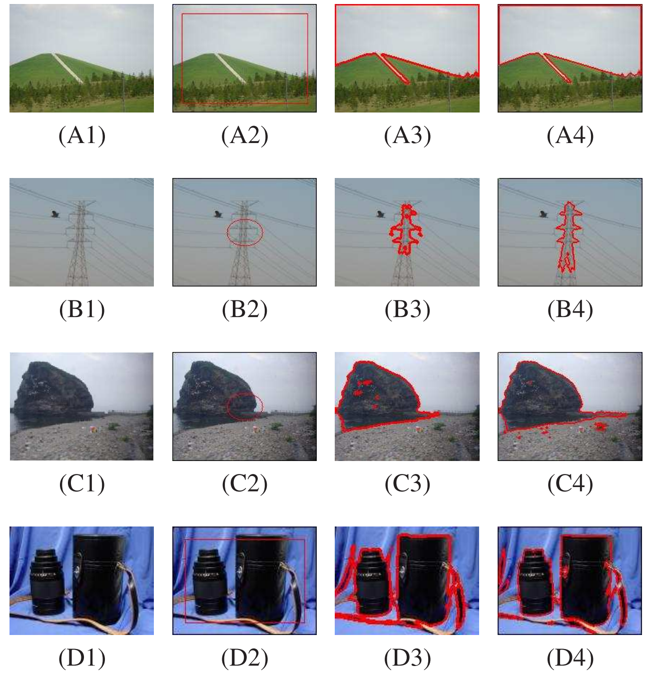

Fig. 6

Comparison of the proposed method and the Chan-Vese model for real images. (A1-D1) Original images. The sizes are: (A1) 200 * 150, (B1) 200 * 150, (C1) 200 * 150, (D1) 149 * 121. (A2-D2) Initial curves. (A3-D3) Segmentation result using the proposed method. The CPU time are: (A3) 0.34s, (B3) 0.124s, (C3) 0.303s, (D3) 0.148s. (A4-D4) Segmentation result using the Chan-Vese model. The CPU time are: (A4) 30.835s, (B4) 20.56s, (C4) 18.316s, (D4)31.142s. The proposed method works almost 100 times faster in these cases.